Skyline queries

Tecnologie e Sistemi per la Gestione di Basi di Dati e Big Data M

Limits of scoring functions

- Although scoring functions are widely used to rank a set of objects, it is nowadays recognized that they have some major **problems**:
  - They have a limited expressive power, i.e., they can only capture those user preferences that "translates into numbers", which is not always the case (or, at least, doing so is not so natural)

    "I prefer having white wine with fish and red wine with meat"
  - Deciding on the "best" scoring function to use and/or the specific weights can be hardly left to the final user, especially when there are several ranking attributes
- In this set of slides we will study an alternative to scoring functions, the so-called skyline queries, that have relevant practical applicability, and also represent a major step towards more general (i.e., powerful) preference models

The concept of tuple dominance

- A fundamental concept underlying the definition of skyline queries is that of

Tuple dominance:

Given a relation R(A1,A2,…,Am,…), in which the Ai's are ranking attributes, assume without loss of generality that on each Ai lower values are better.

A tuple t dominates tuple t' with respect to **A** = {A1,A2,…,Am}, written $\mathbf{t} \succ_{\mathbf{A}} \mathbf{t'}$ or simply $\mathbf{t} \succ \mathbf{t'}$, iff:

$$\forall j = 1,\ldots,m: t.Aj \leq t'.Aj \wedge \exists j: t.Aj < t'.Aj$$

that is:

- t is no worse than t' on all the attributes, and
- strictly better than t' for at least one attribute

Notice that it can well be the case that neither $t \succ t'$ nor $t' \succ t$ hold

- The generalization to the case when the values of some attributes need to be maximized and to arbitrary target points is immediate

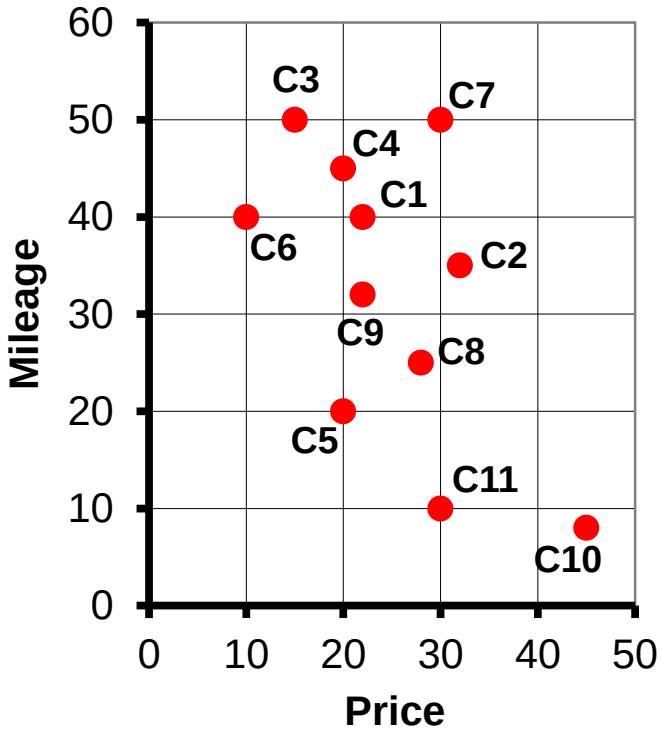
Tuple dominance: example (1)

- Both Points and Rebounds are to be maximized, thus:
  - Tracy McGrady dominates all players but Yao Ming and Shaquille O'Neal
  - Shaquille O'Neal dominates only Yao Ming and Steve Nash
  - Yao Ming dominates only Steve Nash
  - Steve Nash does not dominate anyone
  - …

Name	Points	Rebounds	…
Shaquille O'Neal	1669	760	…
Tracy McGrady	2003	484	…
Kobe Bryant	1819	392	…
Yao Ming	1465	669	…
Dwyane Wade	1854	397	…
Steve Nash	1165	249	…
…	…	…	…

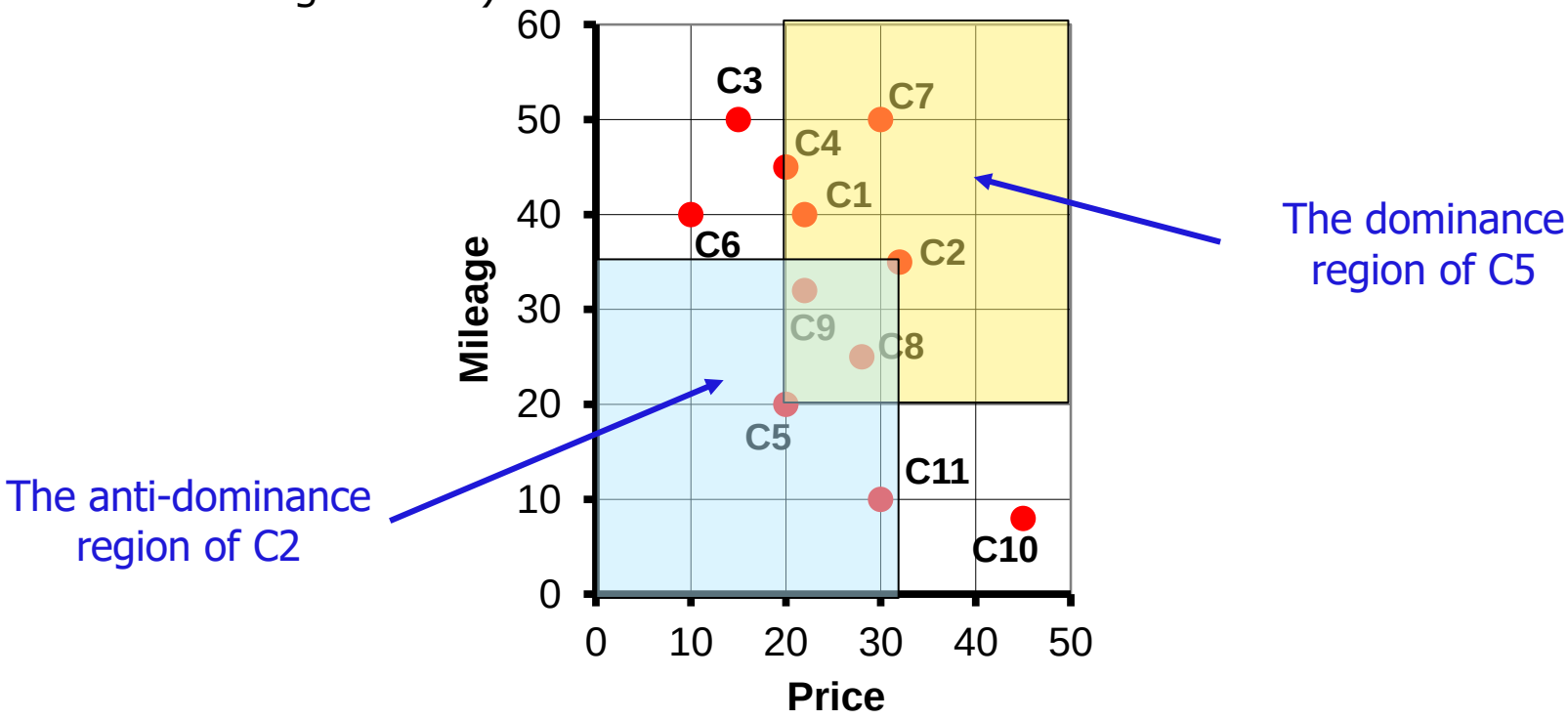
Tuple dominance: example (2)

- Both attributes are to be minimized, thus:
  - Car C6 dominate C1 (same mileage, lower price), C3, C4, and C7
  - Car C5 dominates C1, C2, C4, C7, C8, and C9
  - Car C11 dominates …
  - …



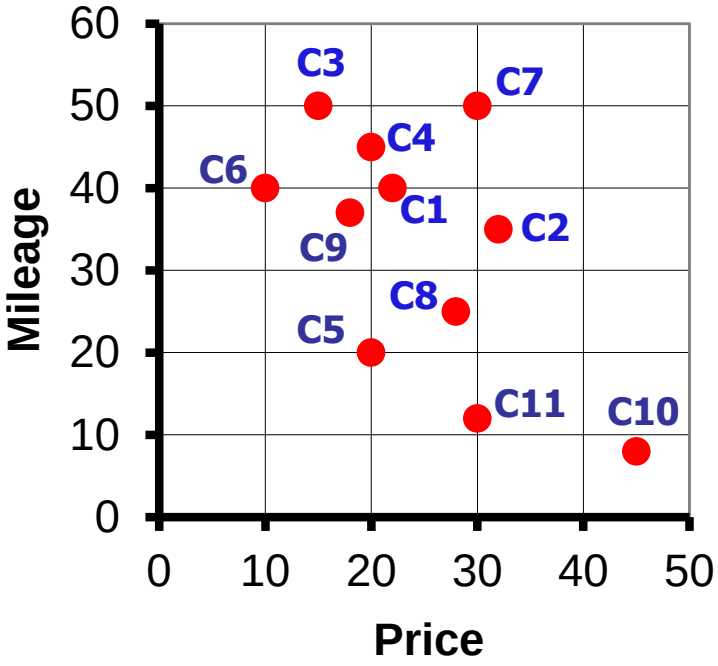
Dominance region

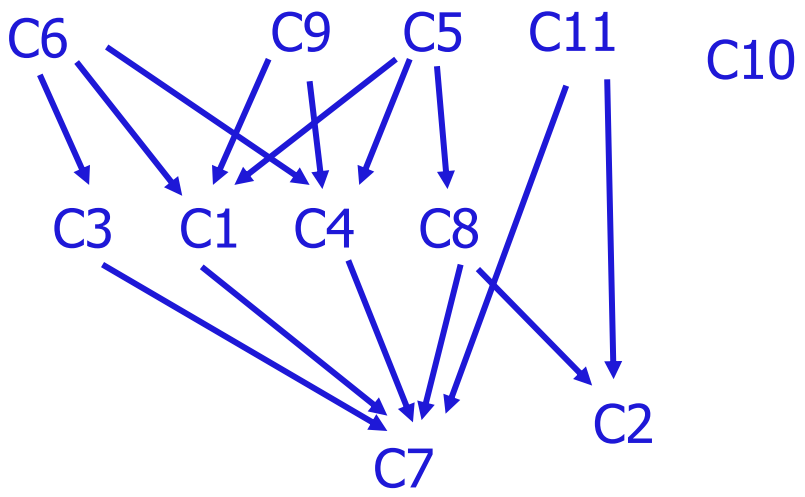
- The dominance region of a tuple t is the set of points in Dom(**A**) that are dominated by t
- Similarly, the anti-dominance region of t is the set of points in Dom(**A**) that dominate t
  - Clearly, t $\succ$ t' iff t' lies in the dominance region of t (and t in the anti-dominance region of t')



The dominance graph

- We omit transitive dominance relationships from the graph (e.g., C6 $\succ$ C7)





Skyline queries

Skyline of a relation [BKS01]:

Given a relation R(A1,A2,…,Am,…), in which the Ai's are ranking attributes, the skyline of R with respect to **A** = {A1,A2,…,Am}, denoted $Sky_{\mathbf{A}}(R)$ or simply Sky(R), is the set of undominated tuples in R:

$$Sky(R) = \{t \mid t \in R, \nexists\, t' \in R: t' \succ t\}$$

- Equivalently, $t \in Sky(R)$ iff no point in R lies in the anti-dominance region of t
- In computational geometry, skyline queries are also known as the "maximal vectors problem"; for multiple criteria optimization problems, their result is a set of so-called Pareto optimal solutions

Skyline queries

Skyline of a relation [BKS01]:

Given a re…e ranking attributes, the skyline of …d $Sky_A(R)$ or simply Sky(R), is the set …

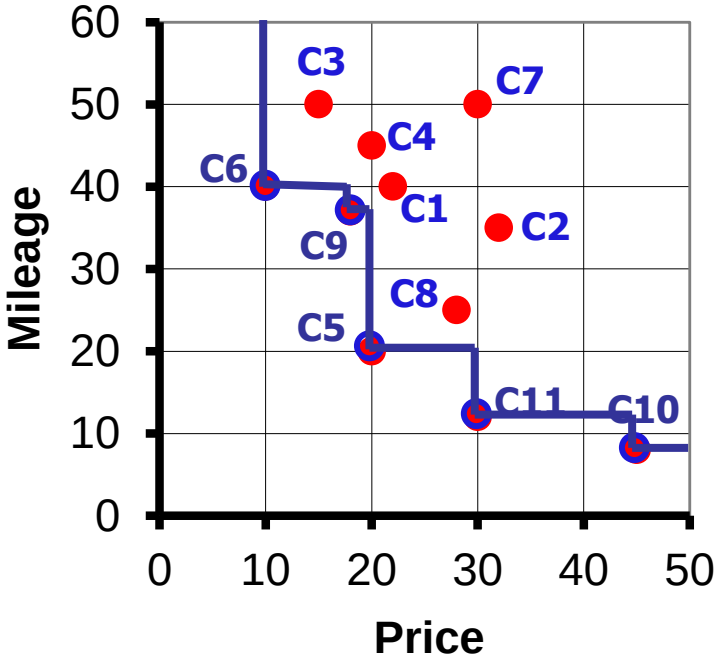
… $\succ t\}$

The Skyline of Manhattan, for instance, can be computed as the set of buildings which are high and close to the Hudson river.
In other words, a building dominates another building if it is higher, closer to the river, and has the same x position [BKS01]

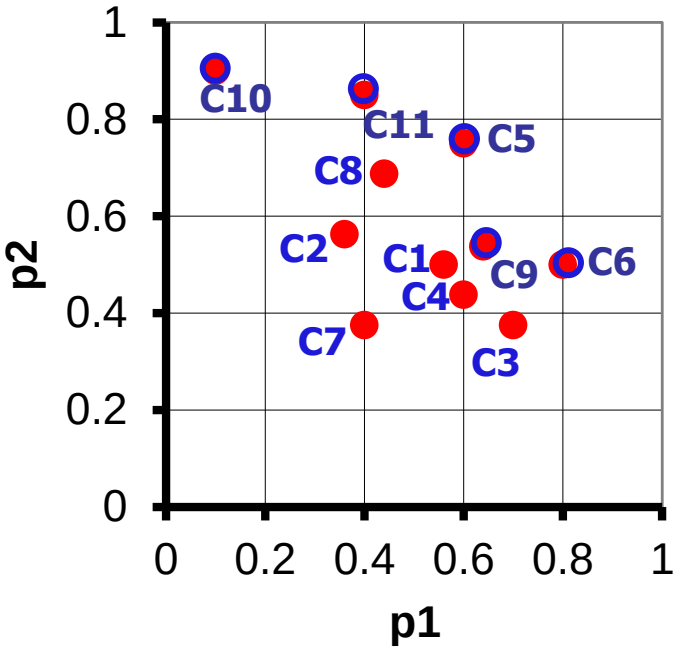
- Equivale…ti-dominance region of t
- In comp…known as the "maximal vectors …problems, their result is a set of so-ca…

A skyline example

- In the attribute space…
  - The "skyline profile" shows the union of the dominance regions of skyline points



- In the score space…
  - No matter how we define scores, the skyline doesn't change!
  - I.e., the skyline is insensitive to any "stretching" of coordinates



What's so special about skyline queries?

- Let **MD** be the set of all monotone distance functions
- We have the following result relating skyline and 1-NN queries, when both have the same target point **q**:

$$t \in Sky(R) \Leftrightarrow \exists\, d \in MD: \forall\, t' \in R, t' \neq t: d(t,q) < d(t',q)$$

- This is to say that:
  1) If t is the (unique) 1-NN for a monotone distance function d, then t is part of the skyline
  2) Conversely, if t is a skyline point, then there exists a monotone distance function d that is minimized by t only

- For this reason, skyline points are also sometimes called "potential NN's"
- Clearly, the same result holds for monotone scoring functions

- **Note**: a non-unique 1-NN is not necessarily undominated (why?)

Proof

1) If t is the unique 1-NN for a monotone distance function d, then t is part of the skyline

- By negating the conclusion.
  Assume t is **not** part of the skyline, i.e., there exists a tuple t' that dominates t.
  For any monotone distance function d it is $d(t',q) \leq d(t,q)$, a contradiction.

2) If t is a skyline point, then there exists a monotone distance function that is minimized by t only

- The proof is constructive. Without loss of generality we can take q = **0**, and assume that all attribute values are strictly positive

  Consider the weighted $\mathbf{L_{\infty,W}}$ distance with weights $w_i = 1/t.Ai$, i=1,…,m.

  It is $L_{\infty,W}(t,\mathbf{0}) = \max_i\{w_i * t.Ai\} = 1$.

  For any other point t' it is is $L_{\infty,W}(t',\mathbf{0}) = \max_i\{w_i * t'.Ai\} = \max_i\{t'.Ai/t.Ai\} > 1$, since t is a skyline point

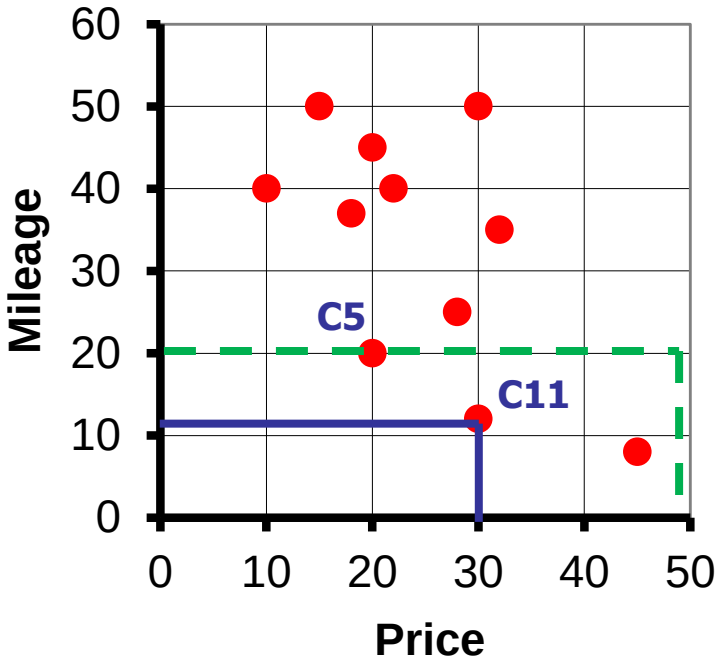
Proof

1) If t is the unique 1-NN for a monotone distance function d, then t is part of the

-

2)

-



C11: (30,12)

$w_1 = 1/30$
$w_2 = 1/12$

$L_{\infty,W}(C11,\mathbf{0}) = 1$

C5: (20,20)

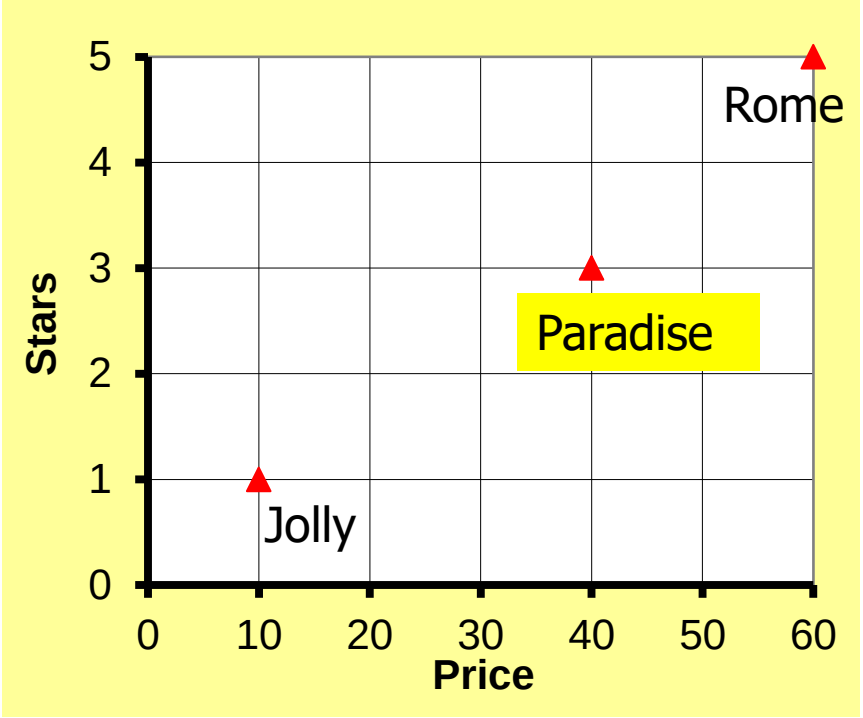
$L_{\infty,W}(C5,\mathbf{0}) = \max\{20/30,20/12\} \approx 1.66$

“Accessibility” of skyline points

S = Ws * Stars – Wp * Price

Hotels

Name	Price	Stars
Jolly	10	1
Rome	60	5
Paradise	40	3



"Accessibility" of skyline points

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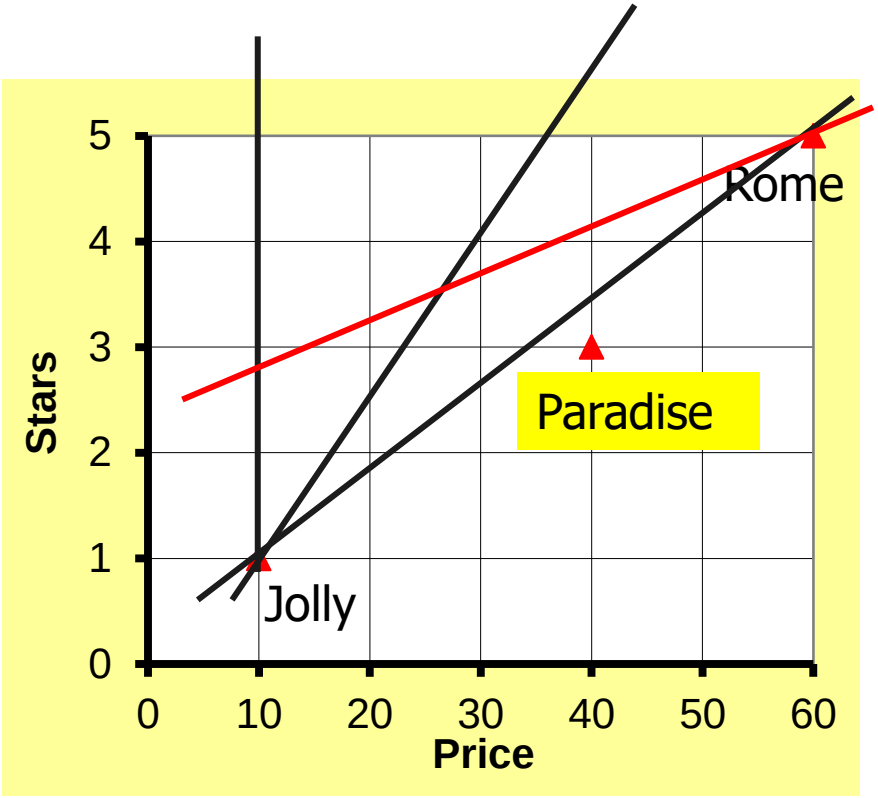


- For no weights combination Paradise is the top-1 hotel
- Similar problems with:
  - Arbitrary values of k and/or
  - Almost all scoring functions

Skylines do not admit any distance function

- The skyline of R does not correspond to any k-NN (or top-k) result, i.e:

Given a schema R(A1,…,Am,…) there is no distance function d (equivalently, scoring function S) that, on all possible instances of R, yields in the first k positions the skyline points

  - Note that here we allow k to be variable, so as to match the actual number of skyline points on each instance of R

Proof: it is Sky(R') = {t1,t4}, thus it has to be: {S(t1), **S(t4)**} **> S(t2)**.

On the other hand, it is Sky(R") = {t2,t3}, thus: {**S(t2)**,S(t3)} **> S(t4)**, a contradiction

R'

TID	p1	p2
t1	0.9	0.6
t2	0.8	0.4
t4	0.5	0.7

R''

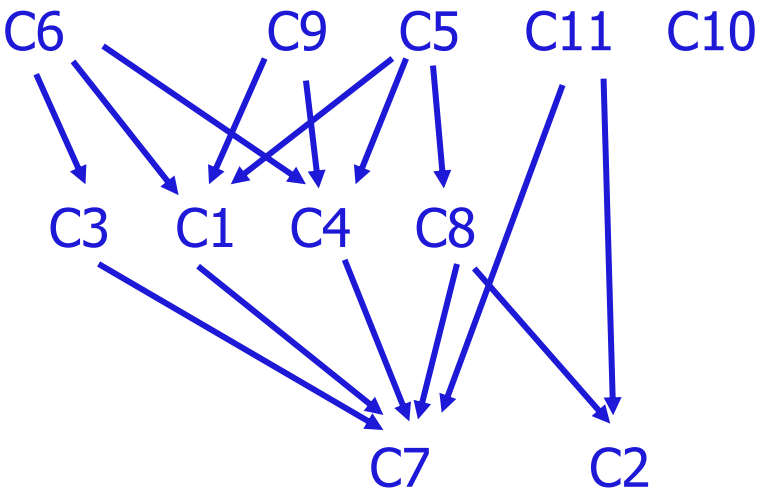
TID	p1	p2
t2	0.8	0.4
t3	0.7	0.8
t4	0.5	0.7

Ranking with skylines

- Ranking of tuples can be easily obtained by iterating the skyline operator
- Define:

$Sky_0(R) = Sky(R)$
$Sky_1(R) = Sky(R - Sky_0(R))$
$Sky_2(R) = Sky(R - Sky_0(R) - Sky_1(R))$
...

- Thus $Sky_0(R)$ are the "top" tuples, $Sky_1(R)$ the "2nd" choices, and so on



$Sky_0(R) = \{C5, C6, C9, C10, C11\}$

$Sky_1(R) = \{C1, C3, C4, C8\}$

$Sky_2(R) = \{C2, C7\}$

Evaluation of skyline queries

- The issue of efficiently evaluating a skyline query has been largely investigated, and many algorithms introduced so far
- A basic reason is that the problem is "more difficult" than top-k queries, since it has a worst-case complexity of $\Theta(N^2)$ for a DB with N objects

- What we see are some algorithms that follow one of the two basic approaches:

Generic:

it computes the skyline without any auxiliary access method (indexes)

- Thus, the input relation can also be the output of some other operation (join, group by, etc.)

Index-based:

it is assumed that an index is available

The naïve Nested-Loops (NL) algorithm

- The simplest (and very inefficient!) way to compute the skyline of R is to compare each tuple with all the others

ALGORITHM NL (nested-loops)

Input: a dataset R, a set of attributes **A** inducing $\succ$

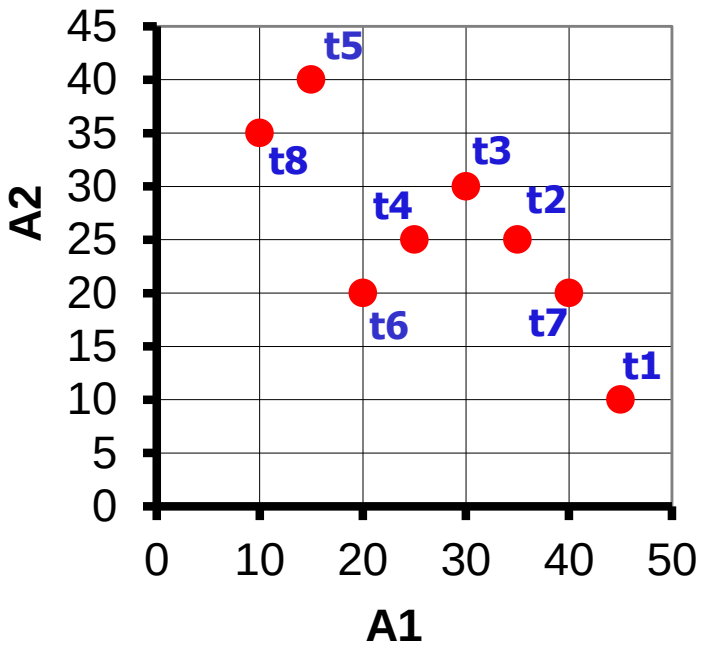
Output: Sky(R), the skyline of R with respect to **A**

```
1.  Sky(R) := ∅;
2.  for all tuples t in R:
3.      undominated := true;
4.      for all tuples t' in R:
5.                  if t' ≻ t then: {undominated := false; break}
6.      if undominated then: Sky(R) := Sky(R) ∪ {t};
7.  return Sky(R);
8.  end.
```

NL: an example

- The origin is the target

R
t1
t2
t3
t4
t5
t6
t7
t8



Sky
t1
t6
t8

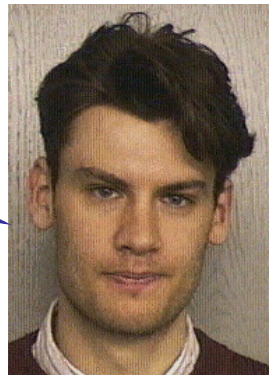
TID	No. of comparisons
t1	7
t2	3
t3	3
t4	5
t5	7
t6	7
t7	6
t8	7
Total	**45**

If t $\in$Sky(R), it will always be compared with all other tuples

The Block-Nested-Loops (BNL) algorithm

- The BNL algorithm [BKS01] improves over NL by immediately discarding all tuples that are dominated by at least one other tuple
- Thus, it also avoids comparing twice the same pair of tuples (as NL does)
- BNL allocates a buffer (*window*) W in main memory, whose size is a design parameter, and sequentially reads the data file
- Every new tuple t that is read from the data file is compared with only those tuples that are currently in W

The BNL algorithm has been proposed in [BKS01] for skyline queries, however its applicability is far more general!



Donald Kossmann

The logic of the BNL algorithm

- When reading a new tuple t, three cases are possible:

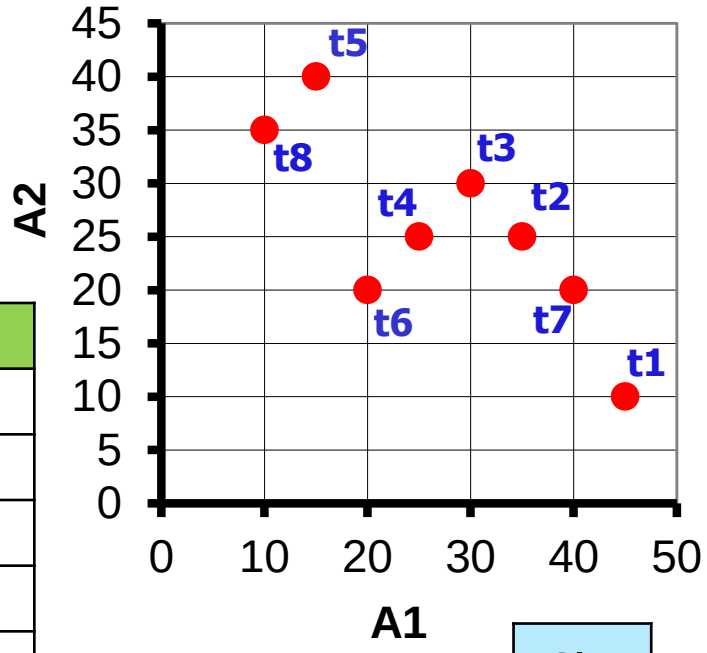
1) If some tuple t' in W dominates t, then t is immediately discarded
2) If t dominates some tuple t' in W, all such tuples are removed from W and t is inserted into W
3) If none of the above two cases holds, then t is inserted into W. However, if no space in W is left, then t is written to a temporary file F

- When all tuples have been processed, if F is empty the algorithm stops, otherwise a new iteration is started by taking F as the new input stream

- The tuples that were inserted in W when F was empty can be immediately output, since they have been compared with all other tuples
- The others in W can be output **during** the next iteration; a tuple t can be output when a tuple t' is found in F that followed t in the sequential order
  - For this, a timestamp (counter) is attached to each tuple

BNL: an example

- Assume |W| = 2
  and the origin as the target



R
t1
t2
t3
t4
t5
t6
t7
t8

1st

W

F
…

Sky

TID	No. of comparisons
t1	7
t2	2
t3	1
t4	2
t5	2
t6	2
t7	0
t8	0
Total	**16**

For each tuple t only comparisons with tuples following t in R are counted

BNL: an example

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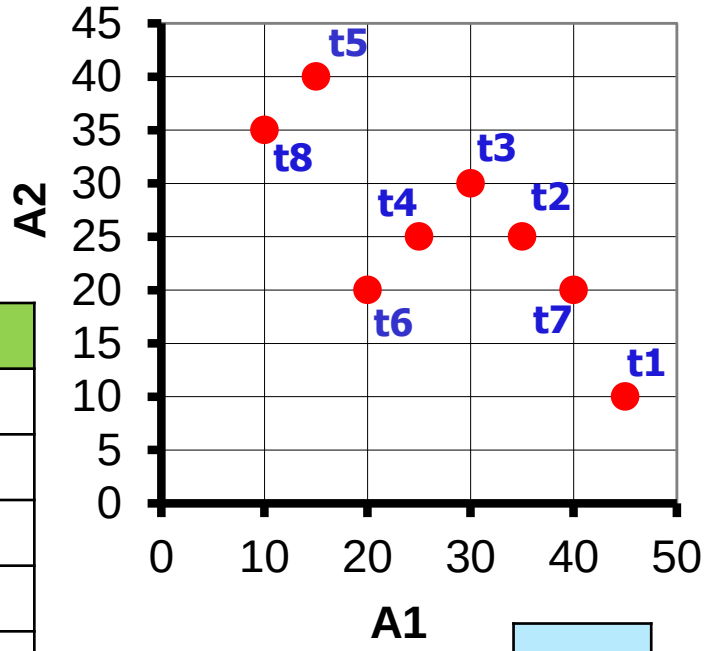

R
t1
t2
t3
t4
t5
t6
t7
t8

1st

W
t1

F
…

Sky

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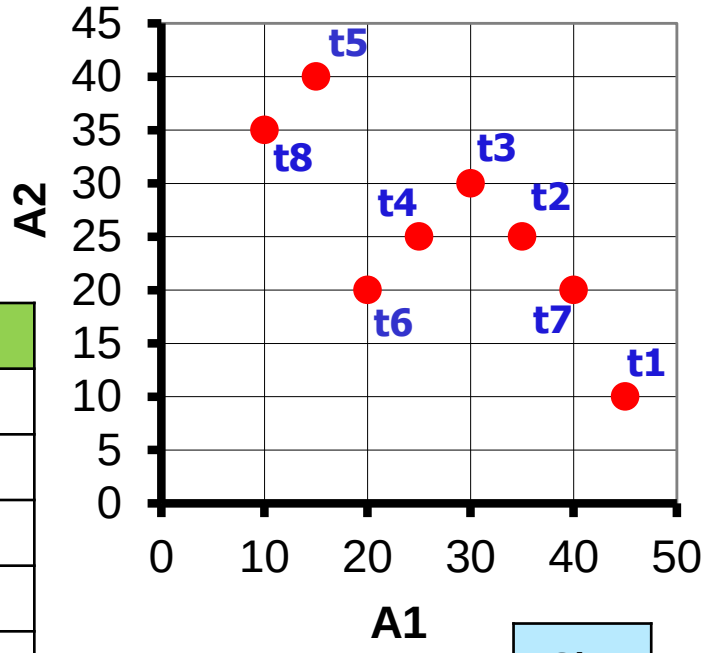

R
t1
t2
t3
t4
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t6
t7
t8

1st

W
t1
t2

F
…

Sky

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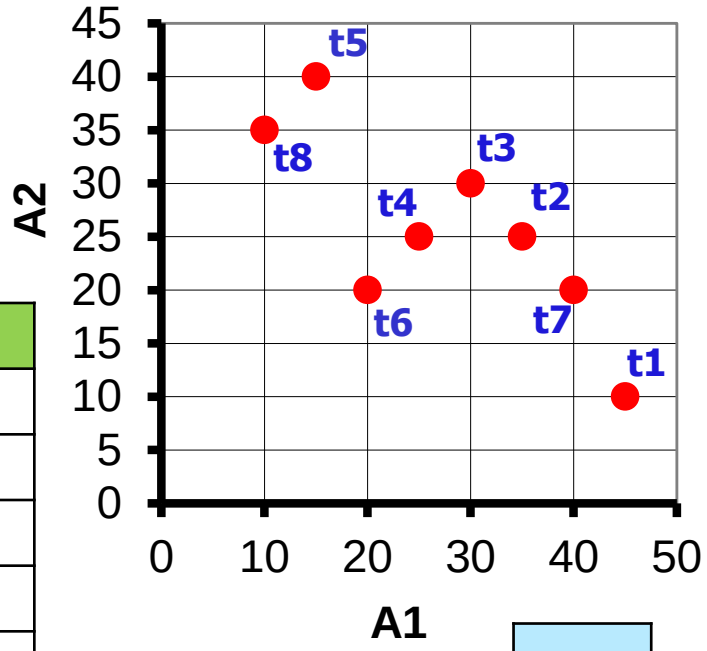

R
t1
t2
t3
t4
t5
t6
t7
t8

1st

W
t1
t2

F
t3
…

Sky

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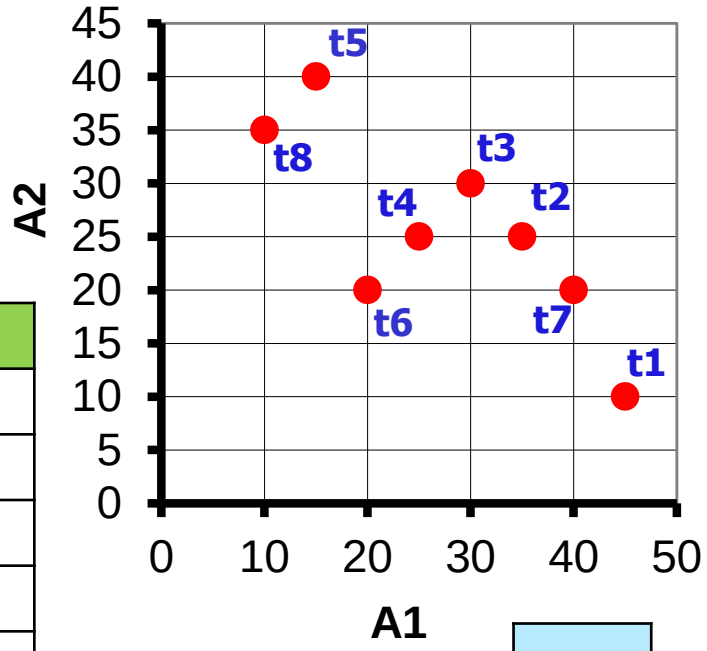

R
t1
t2
t3
t4
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1st

W
t1
t4

F
t3
…

Sky

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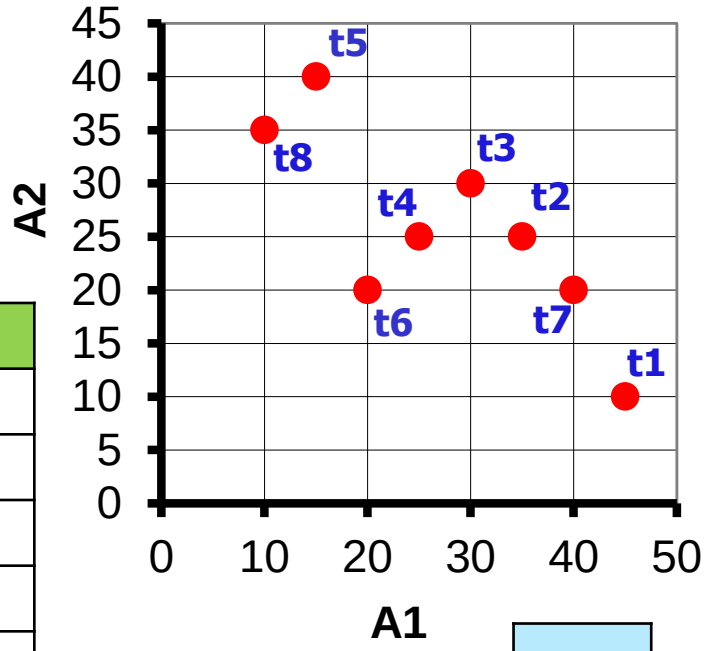

R
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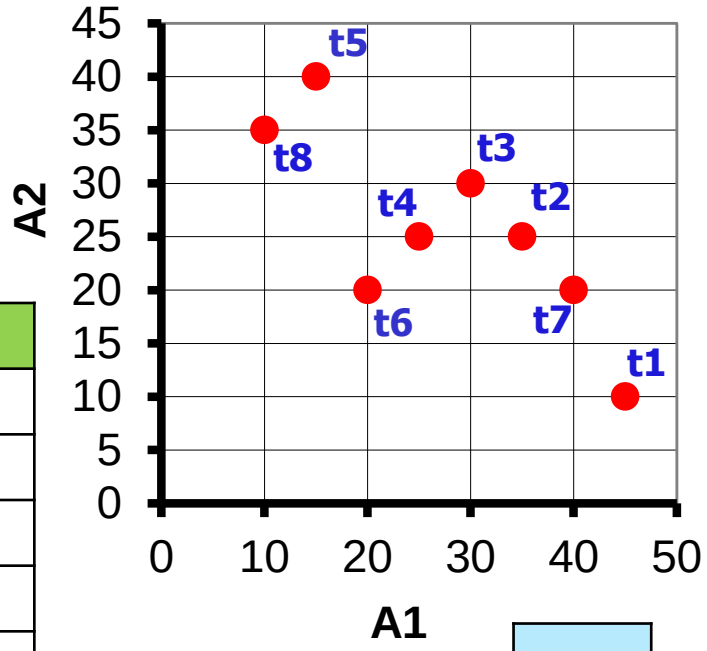

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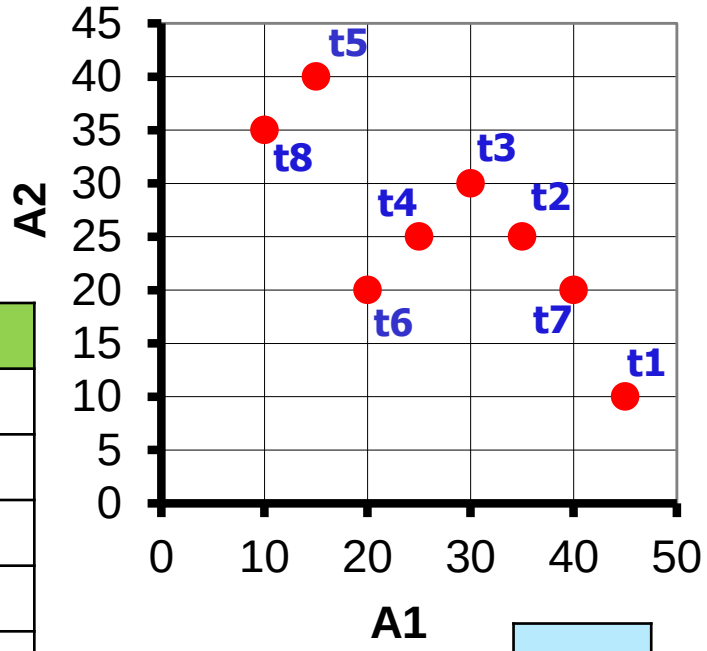

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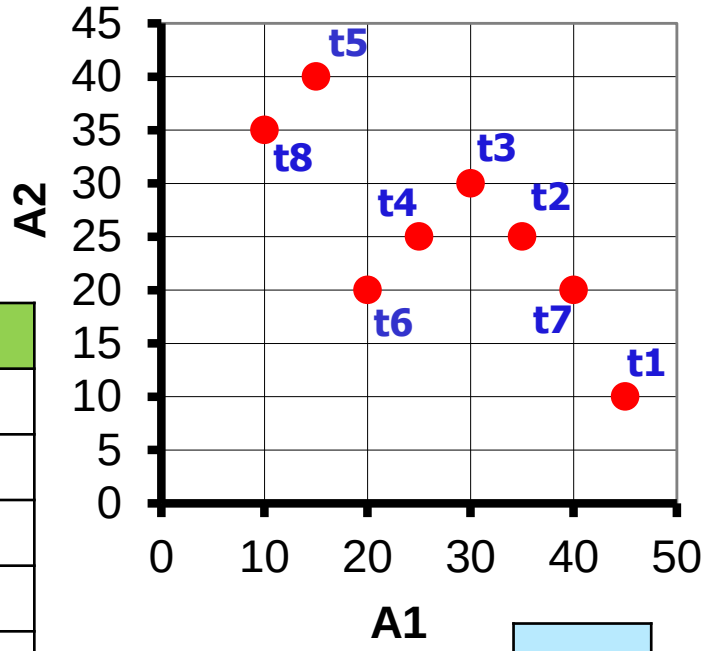

R
t1
t2
t3
t4
t5
t6
t7
t8

Sky
t1

1st

W
t1
t6

F
t3
t5
t8
…

2nd iteration

W
t6

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t1	7
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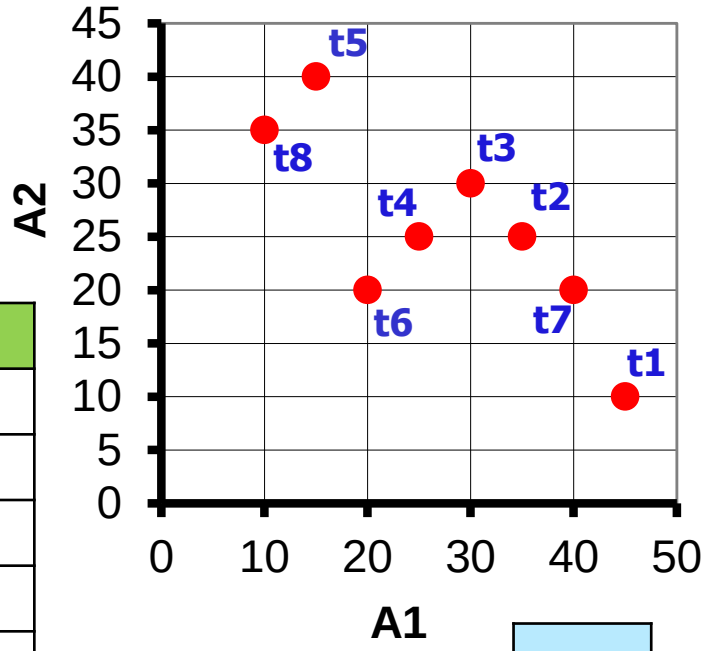

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2nd iteration

W
t6
t5

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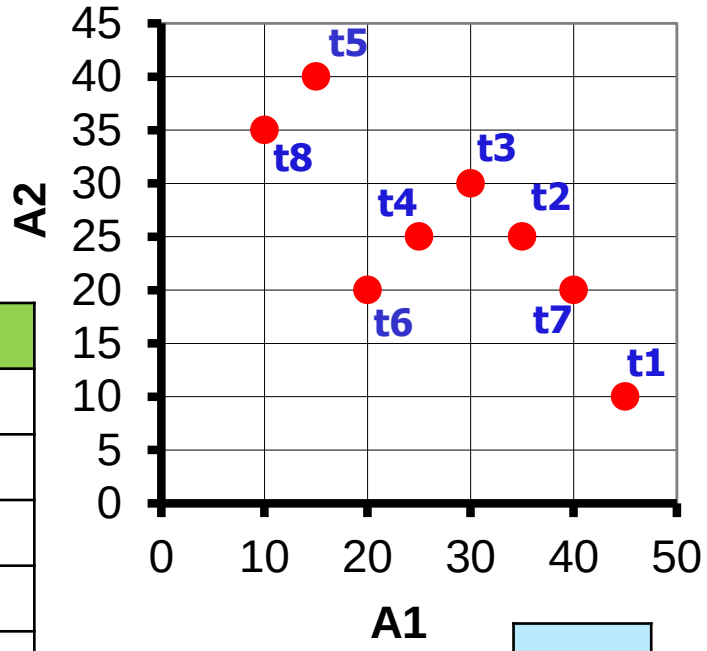
- Assume |W| = 2
  and the origin as the target



R
t1
t2
t3
t4
t5
t6
t7
t8

Sky
t1
t6
t8

1st

W
t1
t6

F
t3
t5
t8
…

2nd iteration

W
t6
t8

TID	No. of comparisons
t1	7
t2	2
t3	1
t4	2
t5	2
t6	2
t7	0
t8	0
Total	**16**

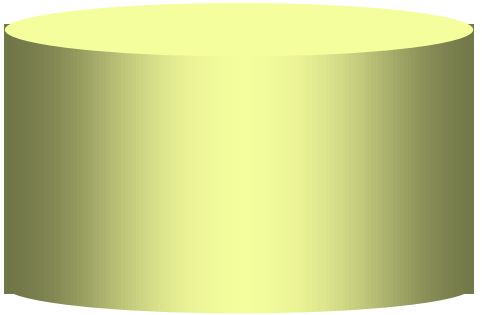
For each tuple t only comparisons with tuples following t in R are counted

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...



F

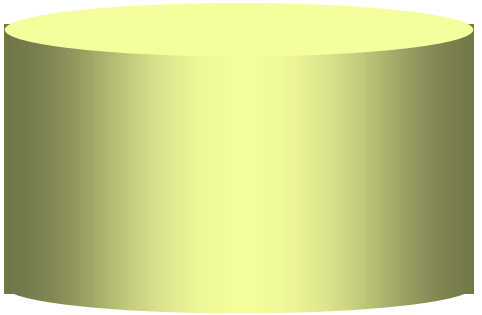
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...
FreshFish	



F

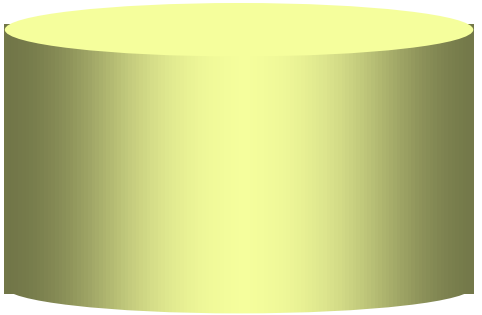
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...
OceanView	



F

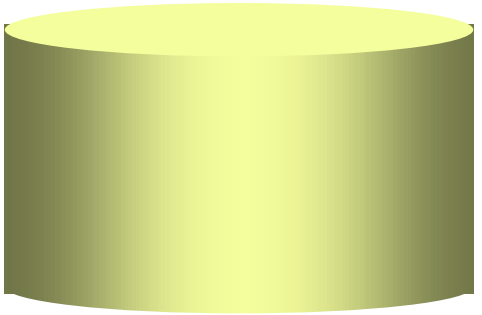
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...
OceanView	
VealHere	



F

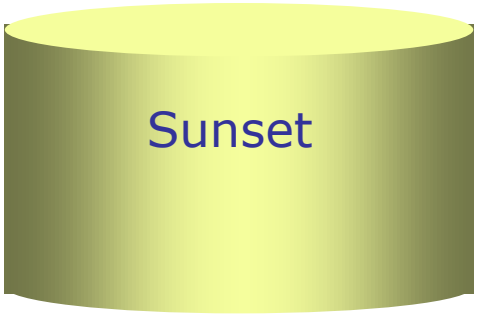
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...
OceanView	
VealHere	



F

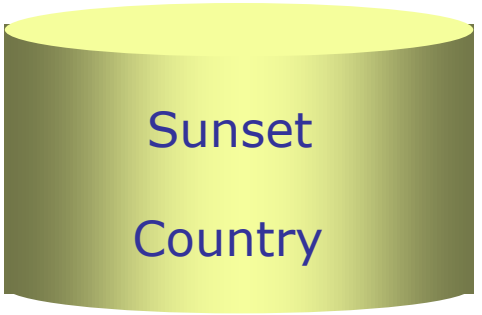
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...
OceanView	
VealHere	



F

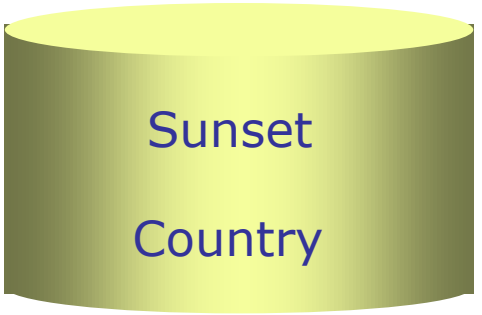
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...
OceanView	
VealHere	



F

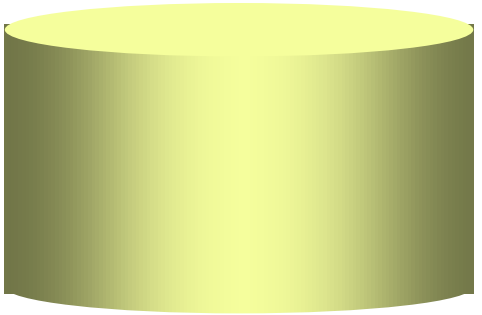
Low(Price) and High(Rating)

BNL: another example

Restaurant	Price	Rating
FreshFish	70	2
OceanView	30	3
VealHere	50	7
Sunset	40	6
Country	48	5
SteakHouse	60	3

W

Restaurant	...



F

OceanView

VealHere

Sunset

Low(Price) and High(Rating)

BNL: some comments

- Experimental results in [BKS01] show that BNL is CPU-bound and that its performance deteriorates if W grows
  - Since with larger W BNL executes more comparisons
- On the other hand, BNL has a relatively low I/O cost

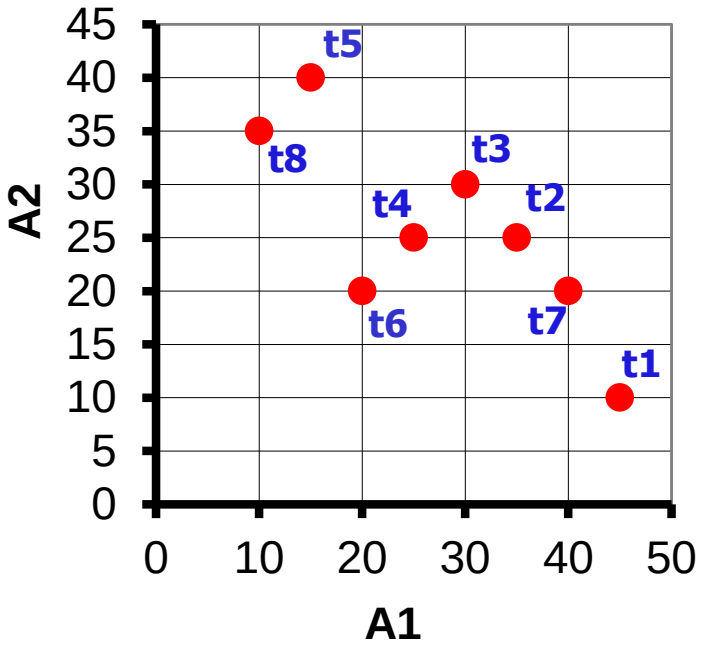
- Performance is also negatively affected by the number of skyline points
- The skyline cardinality depends on the number of attributes and on their correlation
  - Negatively (or anti-)correlated attributes, like Price and Mileage, lead to larger skylines

- [BKS01] also introduces some variants of BNL, among which BNL-sol, that manages W as a self-organizing list
  - The idea is to first compare incoming objects with those in W (called "*killer*" objects) that have been found to dominate several other objects

… and another algorithm (D&C) based on a "divide-and-conquer" approach

BNL: setting |W| = 1

- |W| = 1 yields the minimum number of comparisons for a given input order (equal to those of |W| =2 in this example)



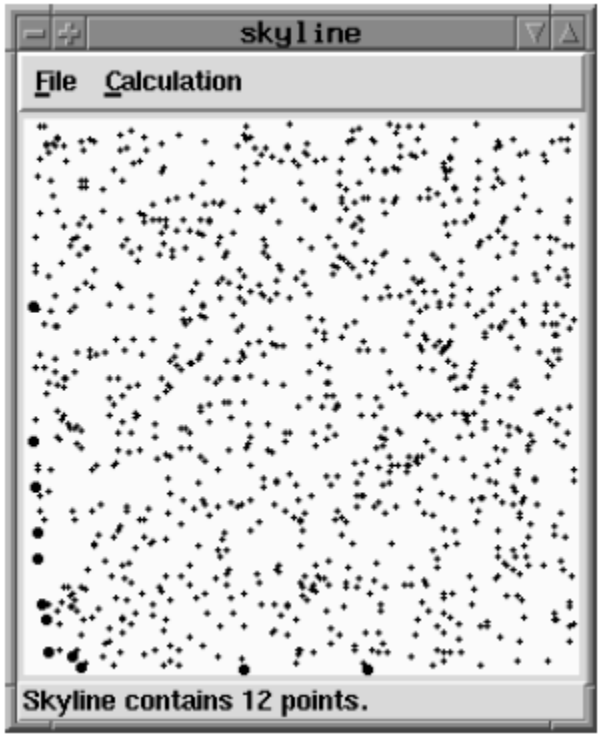
1st	2nd	3rd	3rd (end)
W	W	W	W
t1	t6	t6	t8
F	F	F	F
t2	t3	t5	t5
t3	t5	t8	
t4	t8		
t5			
t6			
t7			
t8			

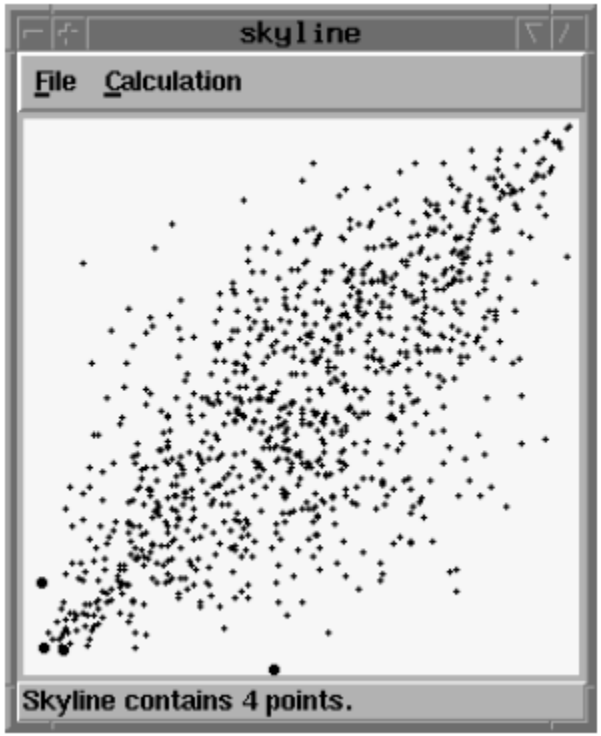
t6 can be output during the 3rd iteration, just after reading t8

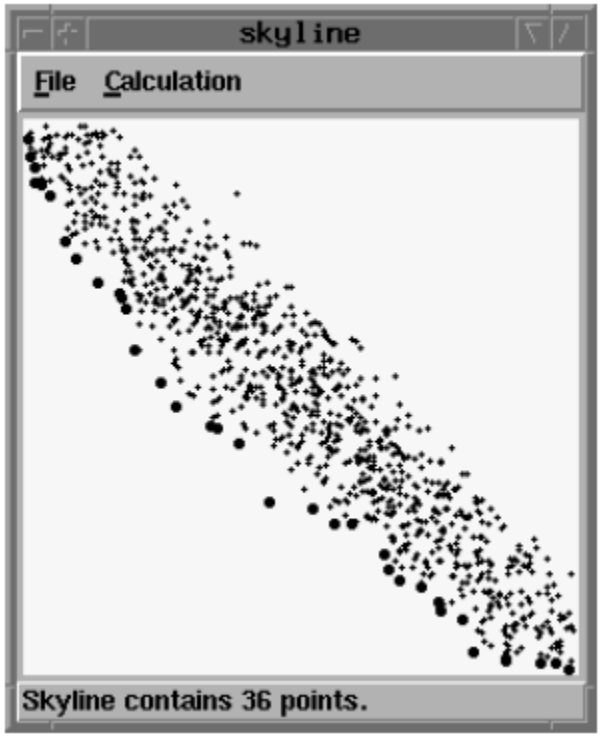
TID	No. of comp.
t1	7
t2	2
t3	1
t4	2
t5	2
t6	2
t7	0
t8	0
Total	**16**

BNL: datasets and experiments (1) [BKS01]

- Synthetic data (uniform independent, correlated and anti-correlated)

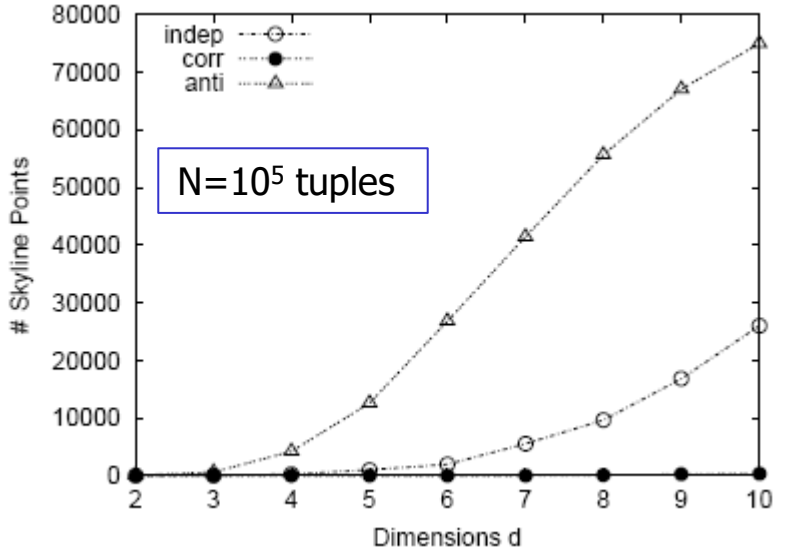


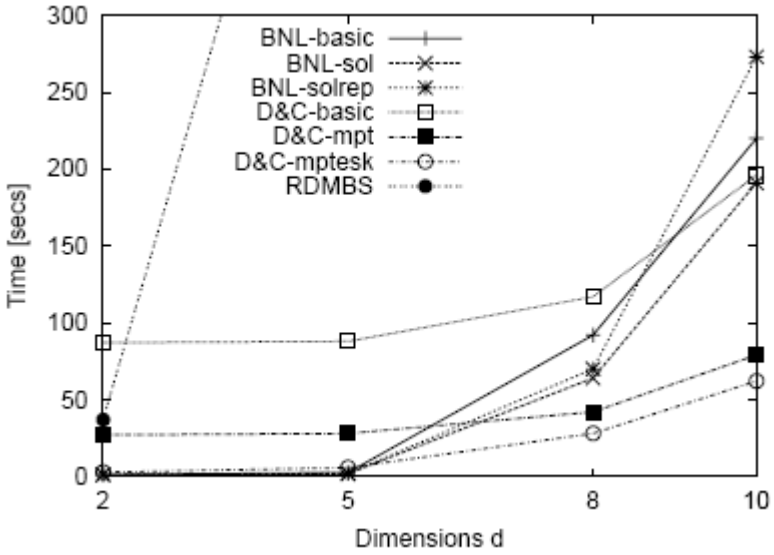




- In the figure: 1000 points (skyline points are in bold)

BNL: datasets and experiments (2) [BKS01]





- RDBMS: the NL algorithm implemented as a correlated subquery:

  "t is part of the skyline if NOT EXISTS(…)"

In this figure:
Independent datasets

dimensionality $\in$ [2,10]
window = 1Mbyte

cardinality N=10^5 tuples

Sun Ultra, 333MHz CPU
128Mbytes RAM

SFS: Sort-Filter-Skyline [CGG+03]

- SFS aims to reduce the overall number of comparisons
- To this end, it first performs a topological sort of the input data, which respects the skyline preference criteria

Topological sort:

Given $\succ$, a topological sort of R is a complete (no ties) ordering < of the tuples in R such that:

$$t \succ t' \Rightarrow t < t'$$

i.e., if t dominates t', then t precedes t' in the complete ordering

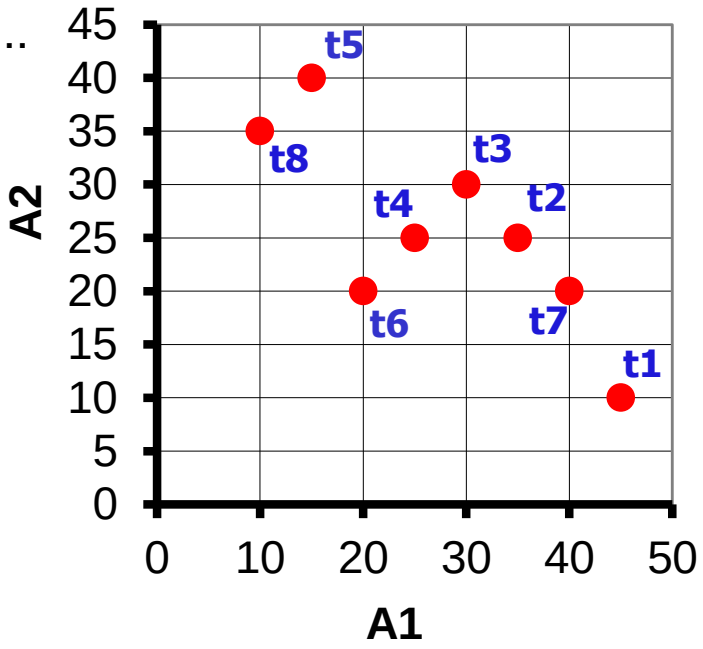
- Here the key observation is:

If the input is topologically sorted,
then a new read tuple cannot dominate
any previously read tuple! ($t > t' \Rightarrow t \not\succ t'$)

Topological sort: example

- For the data in the figure, possible results of a topological sort are:

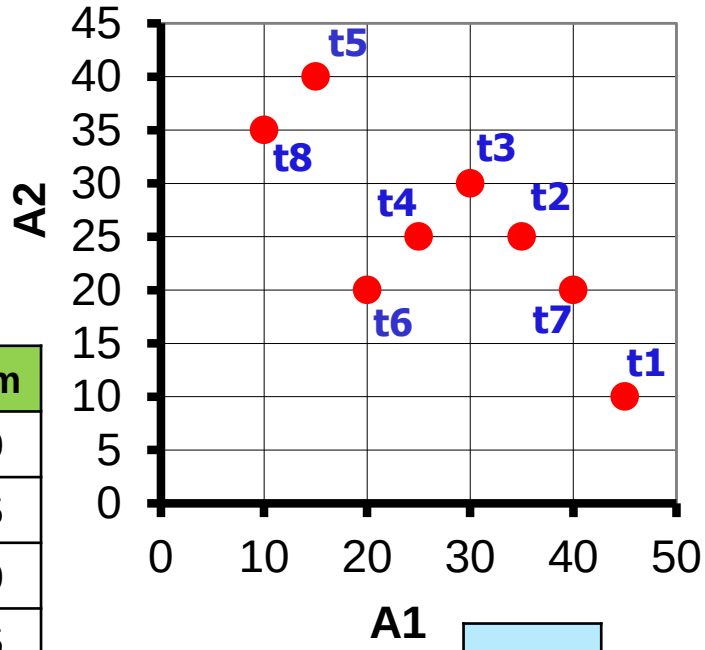
				sum			product	
t6	t8	t1	t6	40	t8	350	...	
t4	t5	t6	t8	45	t6	400		
t2	t6	t4	t4	50	t1	450		
t3	t1	t7	t5	55	t5	600		
t1	t4	t8	t1	55	t4	625		
t8	t7	t3	t2	60	t7	800		
t7	t3	t2	t3	60	t2	875		
t5	t2	t5	t7	60	t3	900		



- In practice, a topological sort is obtained by ordering data using a monotone distance (scoring) function compatible with the skyline criteria

SFS: an example

- Assume |W| = 2
  and the origin as the target



	sum
t6	40
t8	45
t4	50
t5	55
t1	55
t2	60
t3	60
t7	60

1st

W

F
…

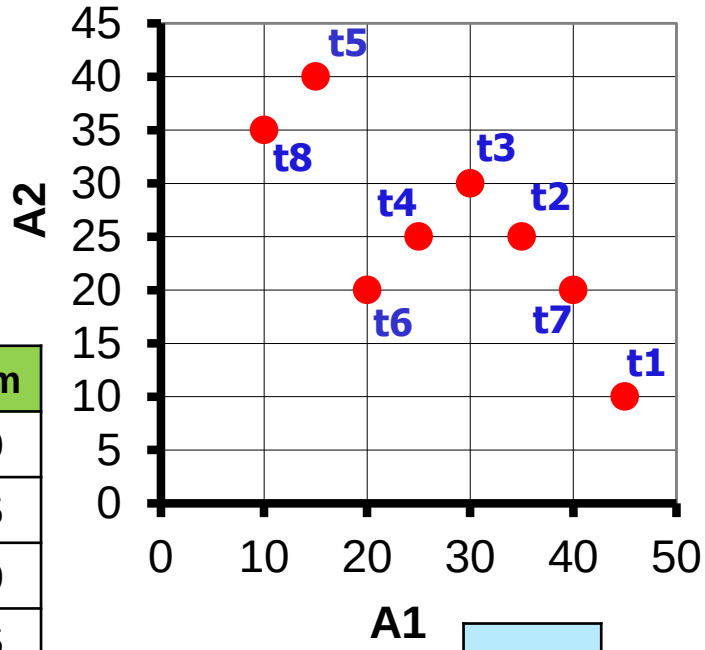
Sky

TID	No. of comp.
t6	7
t8	3
t4	0
t5	0
t1	0
t2	0
t3	0
t7	0
Total	**10**

For each tuple t only comparisons with tuples following t in the sorted input are counted

SFS: an example

- Assume |W| = 2
  and the origin as the target



	sum
t6	40
t8	45
t4	50
t5	55
t1	55
t2	60
t3	60
t7	60

1st

W
t6

F
…

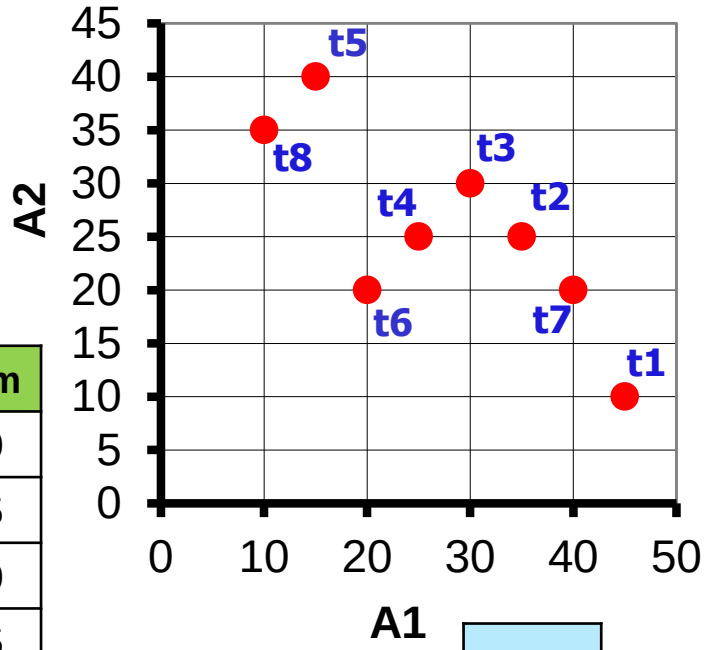
Sky
t6

TID	No. of comp.
t6	7
t8	3
t4	0
t5	0
t1	0
t2	0
t3	0
t7	0
Total	**10**

For each tuple t only comparisons with tuples following t in the sorted input are counted

SFS: an example

- Assume |W| = 2
  and the origin as the target



	sum
t6	40
t8	45
t4	50
t5	55
t1	55
t2	60
t3	60
t7	60

1st

W
t6
t8

F
…

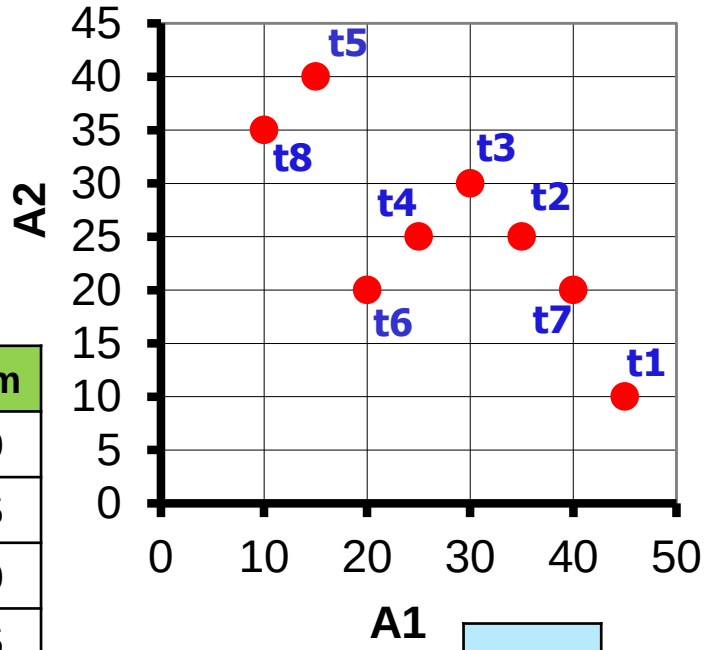
Sky
t6
t8

TID	No. of comp.
t6	7
t8	3
t4	0
t5	0
t1	0
t2	0
t3	0
t7	0
Total	**10**

For each tuple t only comparisons with tuples following t in the sorted input are counted

SFS: an example

- Assume |W| = 2
  and the origin as the target



	sum
t6	40
t8	45
t4	50
t5	55
t1	55
t2	60
t3	60
t7	60

1st

W
t6
t8

F
t1
…

2nd iteration

W
t1

Sky
t6
t8
t1

TID	No. of comp.
t6	7
t8	3
t4	0
t5	0
t1	0
t2	0
t3	0
t7	0
Total	**10**

For each tuple t only comparisons with tuples following t in the sorted input are counted

SFS: further properties

- At the end of each iteration all the tuples in W can be output
  - since no tuple in W can be discarded by a subsequent tuple
- The number of iterations is therefore the minimum one: $\lceil |Sky(R)|/|W| \rceil$
  - In contrast, BNL has no such guarantee
- SFS can return a tuple as soon as it is inserted in the window
  - Therefore, in W one can just store the skyline attribute values, which leads to save (much) space
- Two non-skyline tuples will never be compared
  - Since in W only skyline tuples are present
- Managing the window data structure is now much easier
  - Since only insertions are to be supported
  - No deletion of specific tuples, thus no need to manage empty slots

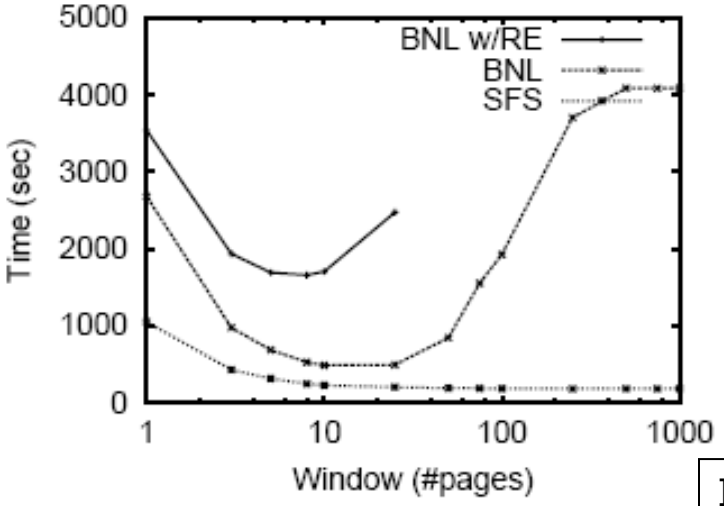
Experimental results (from [CGG+03])

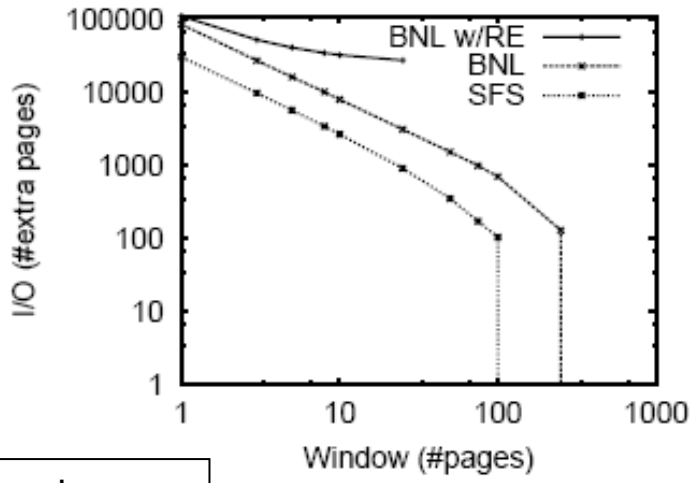
- Data sorted using the "entropy" distance function:

$$d(t,\mathbf{0}) = -\sum_{i=1,m} \ln(2 - t.Ai)$$

$$= -\ln(\exp(\sum_{i=1,m} \ln(2 - t.Ai))) = -\ln\left(\prod_{i=1,m}(2-t.Ai)\right)$$

which yields the same ordering as $2^m - \prod_{i=1,m}(2-t.Ai)$ $(\in [0,2^m - 1])$



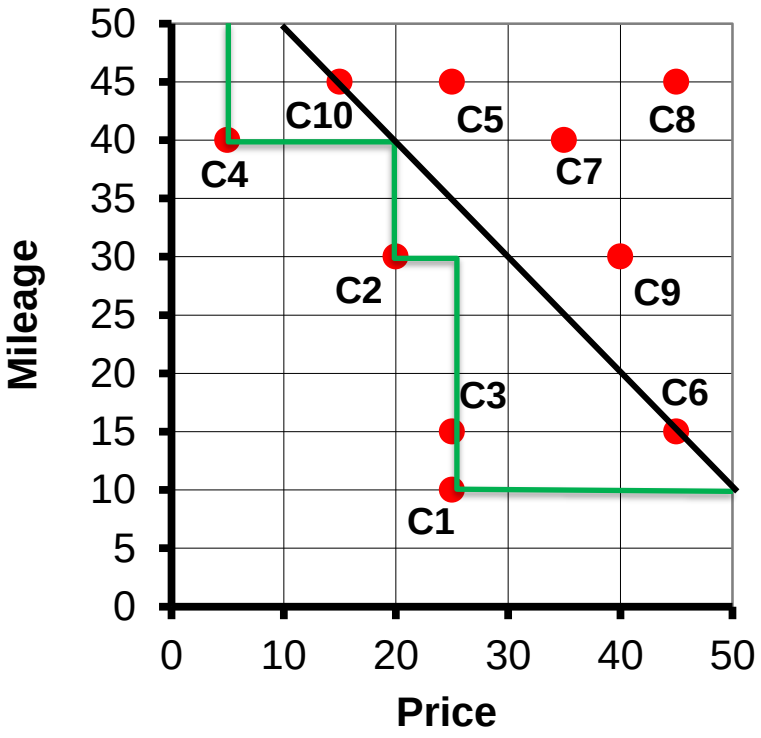


BNL w/RE: input sorted using the "reverse" entropy

Independent dataset
cardinality N=10^6 tuples
dimensionality = 7
window = # 4Kbyte pages
AMD Athlon, 900MHz CPU
384Mbytes RAM

SaLSa [BCP06,BCP08]

- SaLSa (Sort and Limit Skyline algorithm) extends the ideas of SFS by observing that, when data are topologically sorted,
  it is possible to avoid reading all the input tuples



Data sorted using sum:
t.Price + t.Mileage

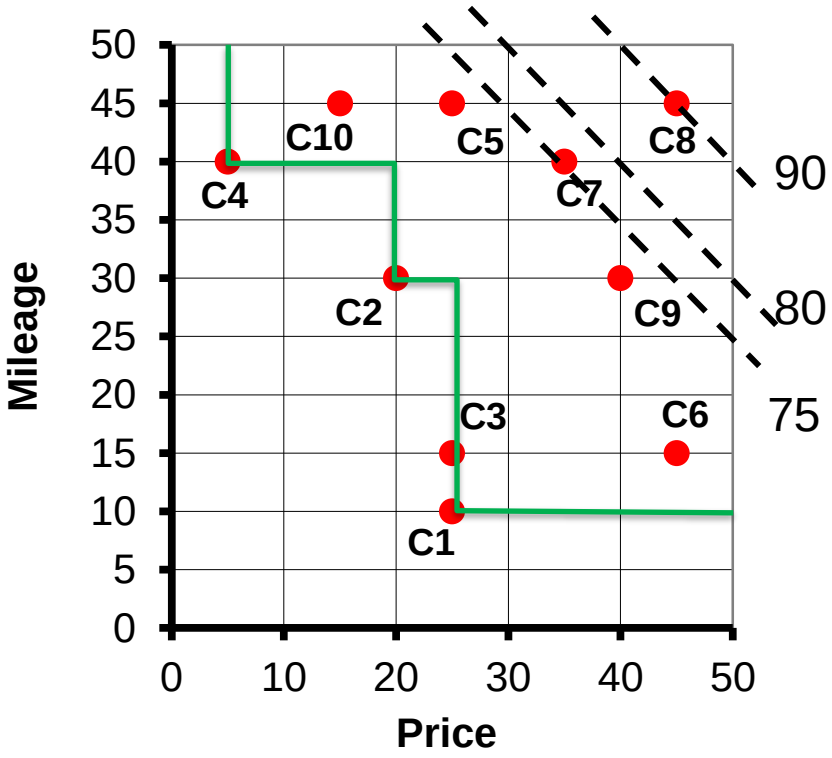
After reading C6 (or C10), whose sum is 60,
we know that no further skyline point exists

… however using **all** the current points in Sky(R) to this purpose is costly:
The problem is NP-hard [BCP08]

And?

The "stop-point"

- SaLSa makes use of a **single** skyline tuple, the so-called stop-point , $\mathbf{t_{stop}}$, to determine when execution can be halted
- In this case it is sufficient to check that what is still to be read lies in the dominance region of t_{stop}



t_{stop}	halt when sum ≥
C1	75
C2	80
C4	90

Choosing the stop-point

- For symmetric distance (scoring) functions, and assuming that on all coordinates the ranges are the same ([0,1], [0,50], etc.) it is possible to prove that the optimal choice for the stop-point is given by the rule:

$$t_{stop} = \text{argmin}_{t \in SKY(R)} \{\max_i \{t.Ai\}\}$$

  that is, the tuple for which the maximum coordinate value is minimum

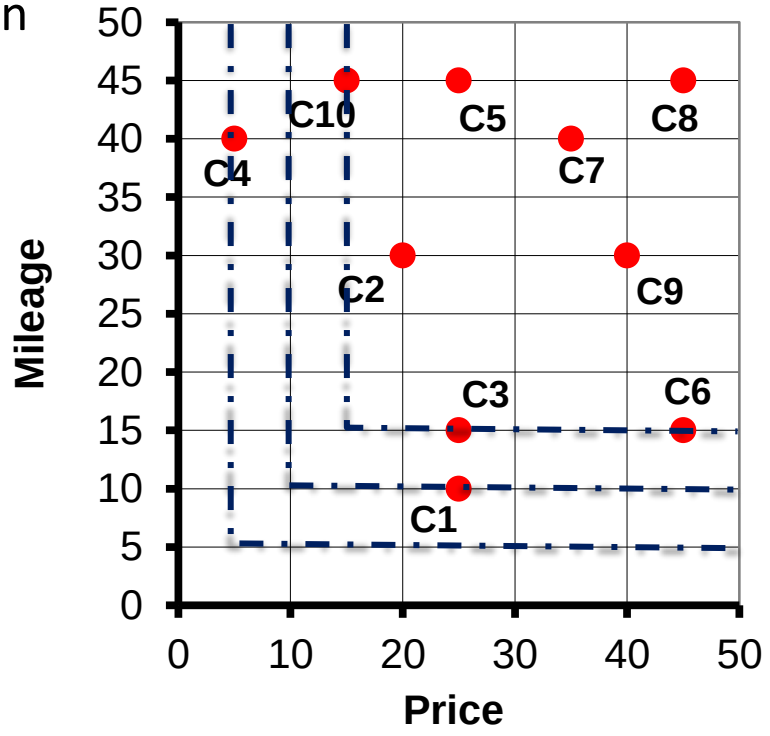
- Note that this holds for **any** symmetric distance function

t_{stop}	Price	Mileage	halt when sum ≥
C1	25	10	75
C2	20	30	80
C4	5	40	90

Optimally ordering the points

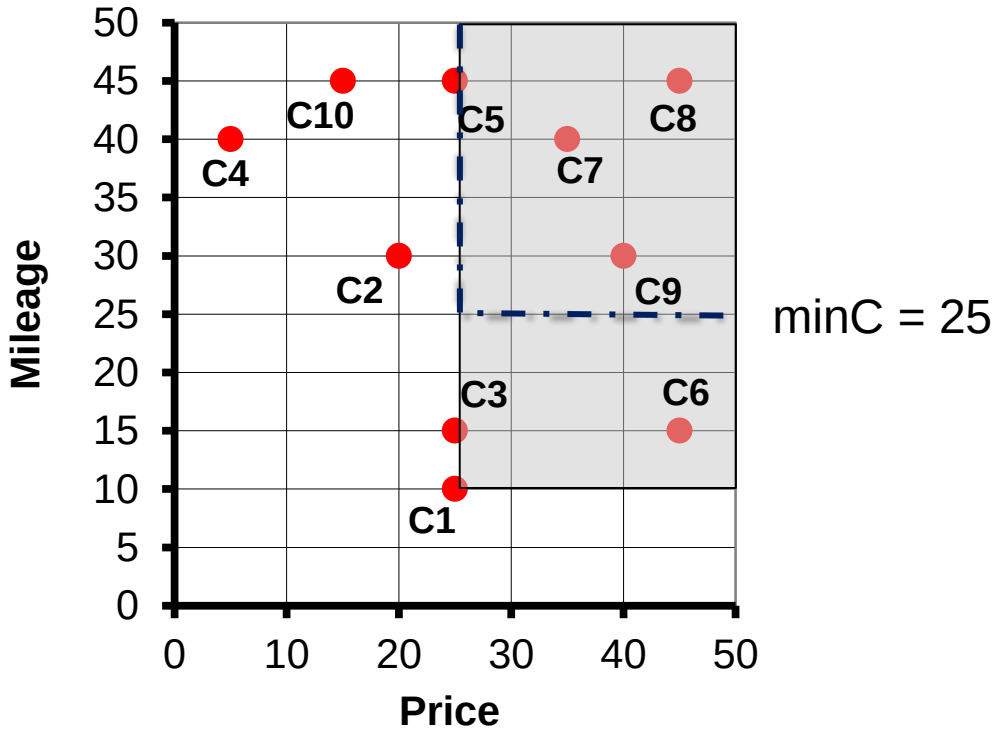
- Among the many alternatives to sort the input data, SaLSa uses a provably optimal criterion, i.e., on each instance ordering data using another (symmetric) function cannot discard more points
- The optimal criterion is called **minC** (minimum coordinate), that is, for each tuple t the value of **$\min_i\{t.Ai\}$** is used
- In case of ties, the secondary criterion "sum" is used

	minC	sum
C4	5	
C1	10	
C3	15	40
C6	15	60
C10	15	60
C2	20	
C5	25	
C9	30	
C7	35	
C8	45	



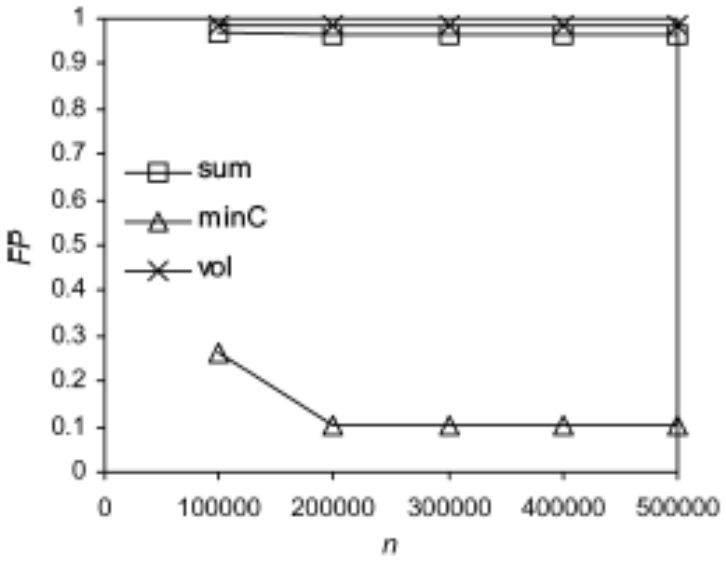
Stopping with minC

- The stop-point is C1, for which it is $\max_i\{C1.Ai\} = 25$
- Thus, as soon as it is minC ≥ 25 SaLSa can be halted
- The general stop condition is therefore: $\min C \geq \max_i\{t_{stop}.Ai\}$

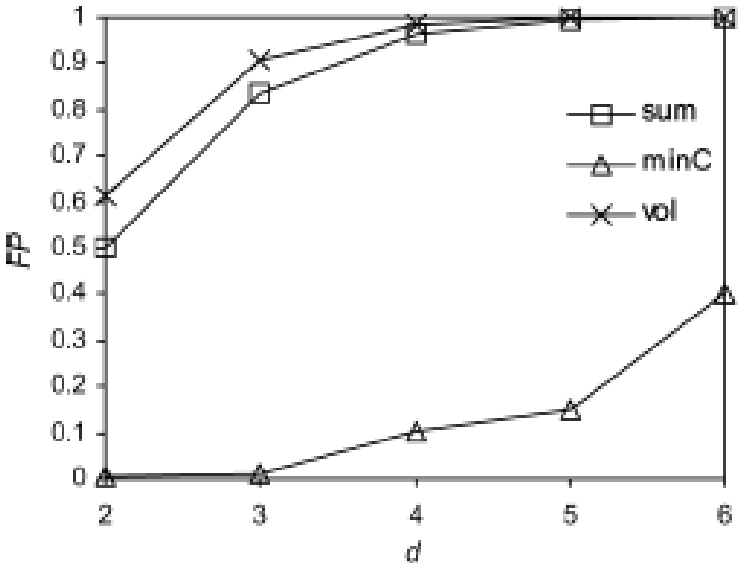


Experimental results (from [BCP08]) (1)

- FP = fraction of fetched points, independent datasets (vol = SFS)



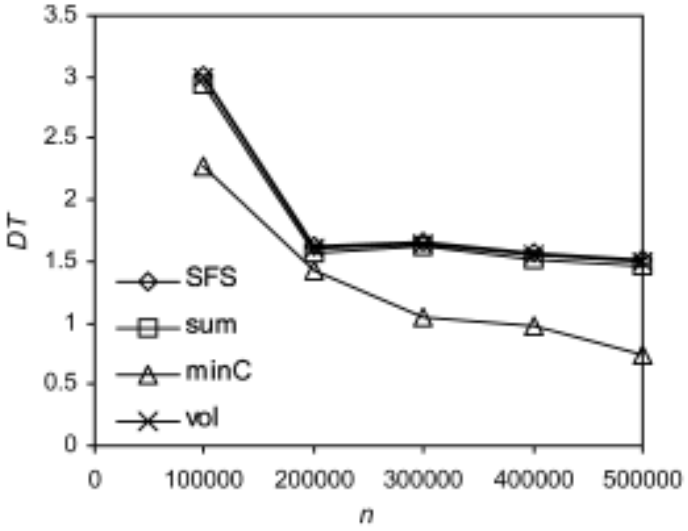
cardinality N $\in$ [10^5,5*10^5] tuples
dimensionality = 4



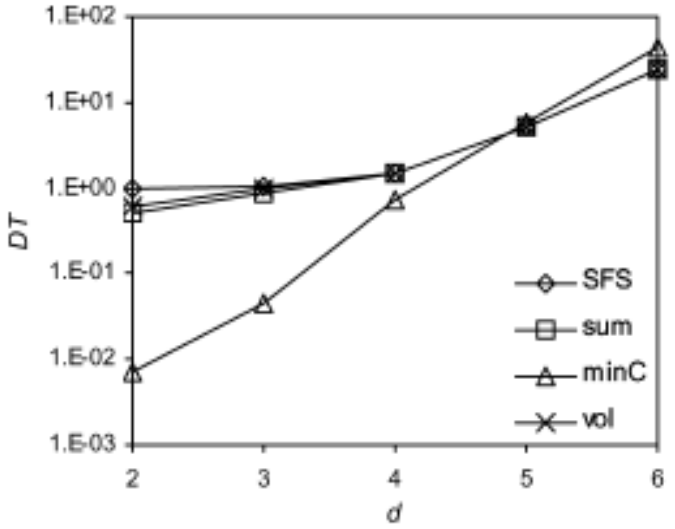
cardinality N=5*10^5 tuples
dimensionality $\in$ [2,6]

Experimental results (from [BCP08]) (2)

- DT = no. of comparisons (dominance tests), normalized to the cardinality of the dataset



cardinality N $\in$ [10^5,5*10^5] tuples
dimensionality = 4



cardinality N=5*10^6 tuples
dimensionality $\in$ [2,6]

Experimental results (from [BCP08]) (3)

- Mixed dataset = half points are anti-correlated, others are dominated

Table II. Elapsed Time on Synthetic Datasets ($n = 500K$, $d = 4$. Times are in Seconds)

	Uniform			Mixed		
	sum	minC	vol	sum	minC	vol
Sorting	1.57	1.79	1.64	2.15	2.40	2.44
Fetching	3.85	0.42	3.94	3.82	1.91	3.90
Filtering	2.11	0.10	2.21	5.02	2.57	6.26
Total	7.53	2.31	7.79	10.99	6.88	12.60

cardinality N = 10^5 tuples
dimensionality = 4
Data stored and sorted in IBM DB2
Pentium IV, 3.4GHz CPU
512Mbytes RAM

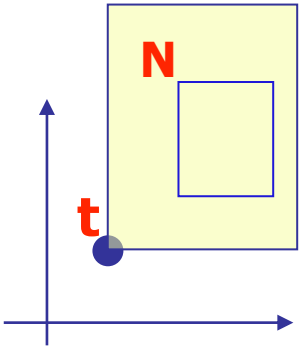
Table III. Elapsed Time on Real Datasets ($d = 4$)

	NBA			Color			Household			EEG		
	sum	minC	vol	sum	minC	vol	sum	minC	vol	sum	minC	vol
Sorting	0.02	0.03	0.02	0.07	0.11	0.07	0.34	0.40	0.36	39.97	48.65	40.33
Fetching	0.08	0.03	0.09	0.54	0.08	0.55	1.02	0.01	1.02	22.59	0.01	22.59
Filtering	0.03	0.01	0.04	0.35	0.06	0.34	0.52	0.01	0.52	23.39	0.01	24.46
Total	0.13	0.07	0.15	0.96	0.25	0.96	1.88	0.42	1.90	85.95	48.67	87.38

Computing the skyline with R-trees

- If we have an index over the ranking attributes, we can use it to avoid scanning the whole DB
- The BBS (Branch and Bound Skyline) algorithm [PTF+03] is reminiscent of kNNOptimal, in that it accesses index nodes by increasing values of MinDist (in the following the query/target point coincides with the origin) and of next-NN, in that the queue PQ keeps both tuples and nodes
  - For computational economy, [PTF+03] evaluates distances using L_1 (Manhattan distance)
- The basic objective of the algorithm is to avoid accessing index nodes that cannot contain any skyline object
- To this end it exploits the following simple observation:

  If the region Reg(N) of node N completely lies in the dominance region of a tuple t, then N cannot contain any skyline point ("t dominates N")



- It also exploits the (now well-known) fact that **if $L_1(t',0) \geq L_1(t,0)$ then $t' \not\succ t$**
- PQ also stores key(N), i.e., the MBR of N, in order to check if N is dominated by some tuple t

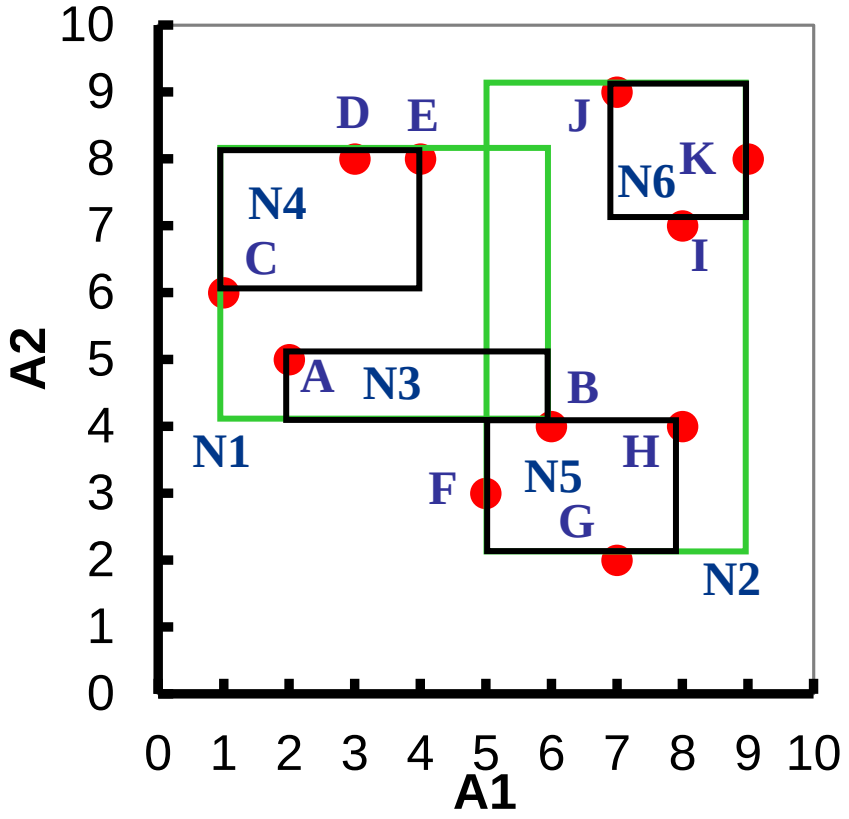
The BBS algorithm

Input: index tree with root node RN
Output: Sky, the skyline of the indexed data

```
1.  Initialize PQ with [ptr(RN),Dom(R),0];    // starts from the root node
2.  Sky := ∅;                  // the Skyline is initially empty
3.  while PQ ≠ ∅:                          // until the queue is not empty…
4.      [ptr(Elem), key(Elem), dMIN(0,Reg(Elem))] := DEQUEUE(PQ);
5.      If no point in Sky dominates Elem then:
6.          if Elem is a tuple t then: Sky := Sky ∪ {t}
7.              else: { Read(Elem);   // …node Elem might contain skyline points
8.                  if Elem is a leaf then: { for each tuple t in Elem:
9.                      if no tuple in Sky dominates t then:
10.                                  ENQUEUE(PQ,[ptr(t), key(t), L1(0,key(t))]) }
11.                              else: { for each child node Nc of Elem:
12.                     if no point in Sky dominates Nc then:
13.                             ENQUEUE(PQ,[ptr(Nc), key(Nc), dMIN(0,Reg(Nc))]) }};
14. return Sky;
15. end.
```

BBS: An example (1/2)

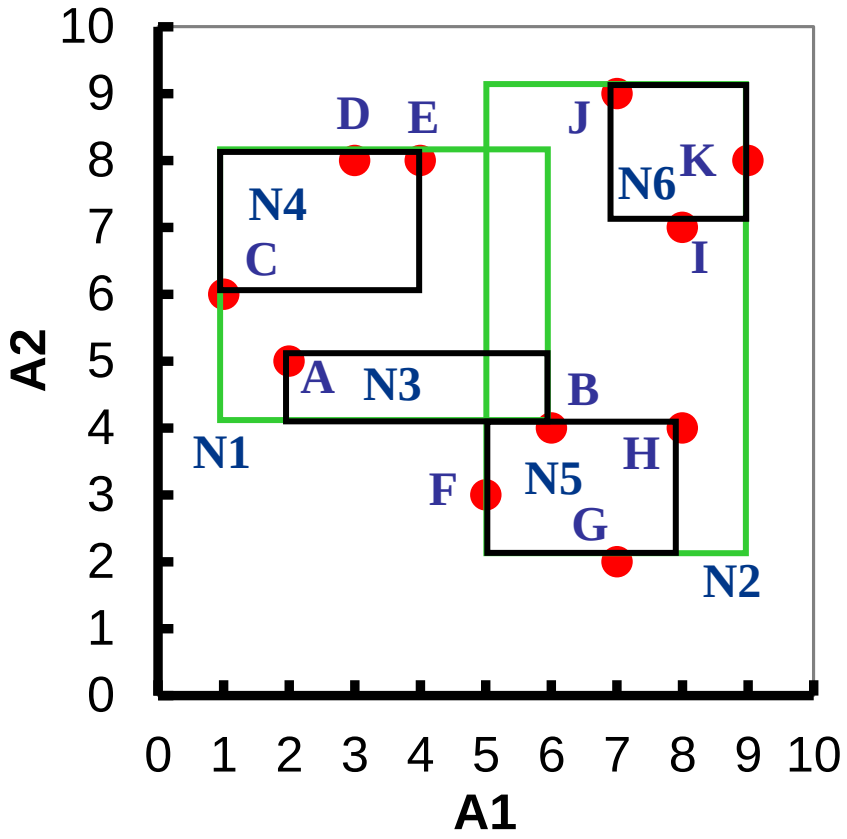
- distance: L1



Elem	d_{MIN}
A	7
B	10
C	7
D	11
E	12
F	8
G	9
H	12
I	15
J	16
K	17
N1	5
N2	7
N3	6
N4	7
N5	7
N6	14

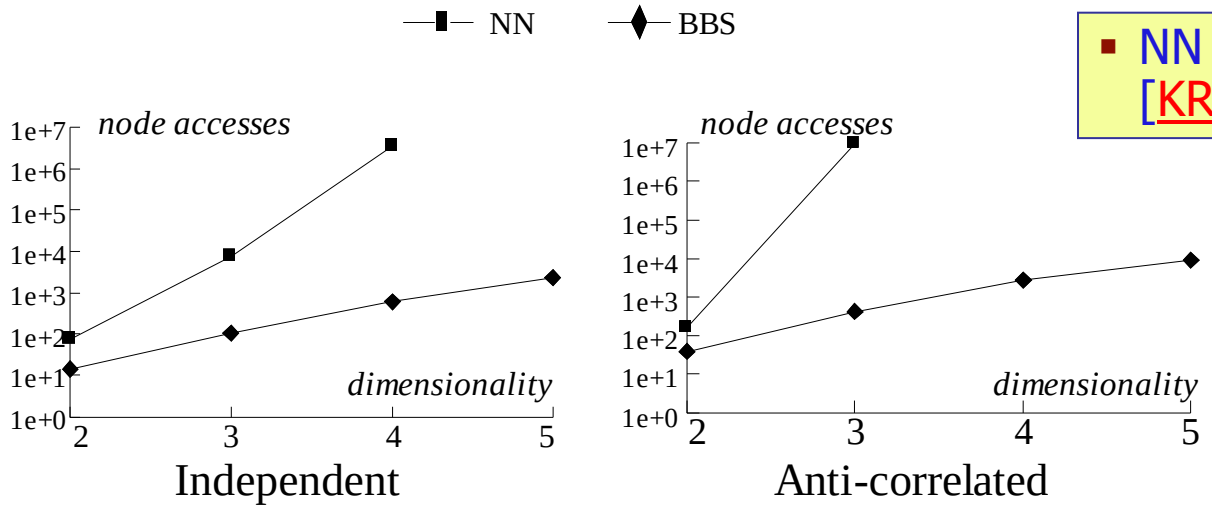
BBS: An example (2/2)

Action	PQ			
Read(RN)	(N1,5)	(N2,7)		
Read(N1)	(N3,6)	(N4,7)	(N2,7)	
Read(N3)	(A,7)	(N4,7)	(N2,7)	(B,10)
Return(A)	(N4,7)	(N2,7)	(B,10)	
Read(N4)	(C,7)	(N2,7)	(B,10)	
Return(C)	(N2,7)	(B,10)		
Read(N2)	(N5,7)	(B,10)		
Read(N5)	(F,8)	(G,9)	(B,10)	
Return(F)	(G,9)	(B,10)		
Return(G)	(B,10)			

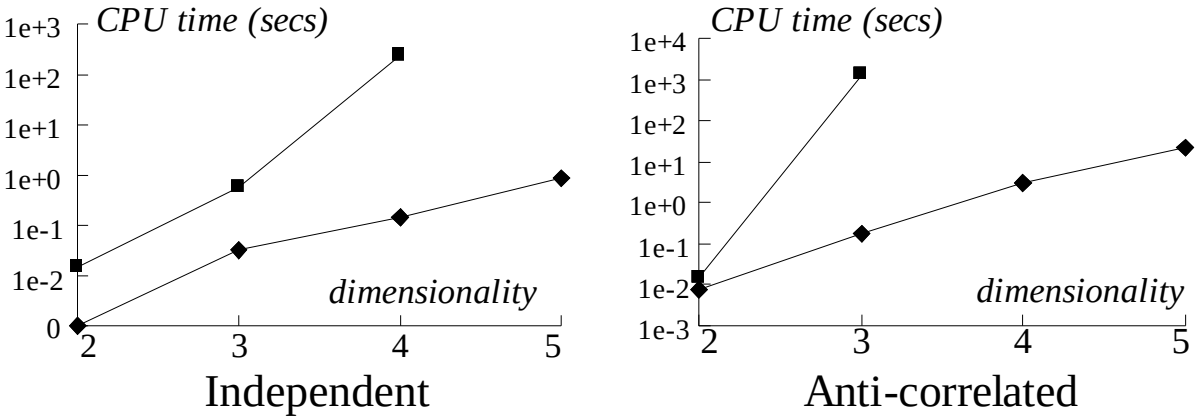


- The example clearly shows why a tuple currently undominated, such as B, which is stored in N3, needs to be inserted into the queue

Experimental results (from [PTF+03])



Node accesses vs. d (N=1M)



CPU-time vs. d (N=1M)

- NN is an algorithm from [KRR02], also based on R-trees

Experimental setup
Independent (uniform) and anti-correlated datasets

dimensionality $\in$ [2,5]
cardinality N=1M tuples

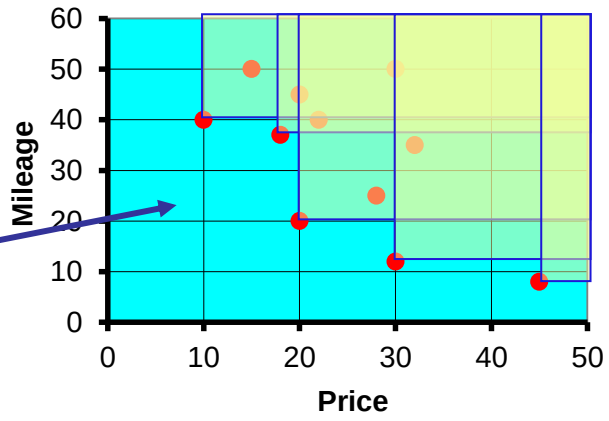
Node size = 4Kbytes
(C = 204 when d=2;
C = 94 when d=5)

Pentium 4, 2.4GHz CPU
512Mbytes RAM

Correctness and optimality of BBS

- The correctness of BBS is easy to prove, since the algorithm only discards nodes that are found to be dominated by some point in the Skyline
- As SFS and SaLSa, when a tuple t is inserted into Sky, then t is guaranteed to be part of the final result
  - This is a direct consequence of accessing nodes by increasing values of MinDist and of inserting a tuple into Sky only when it becomes the first element of PQ
- Optimality of BBS (which we do not formally prove) means:
  BBS only reads those nodes that intersect the "Skyline search region"; this is the complement of the union of the dominance regions of skyline points

The Skyline search region



Skylines for low-cardinality domains

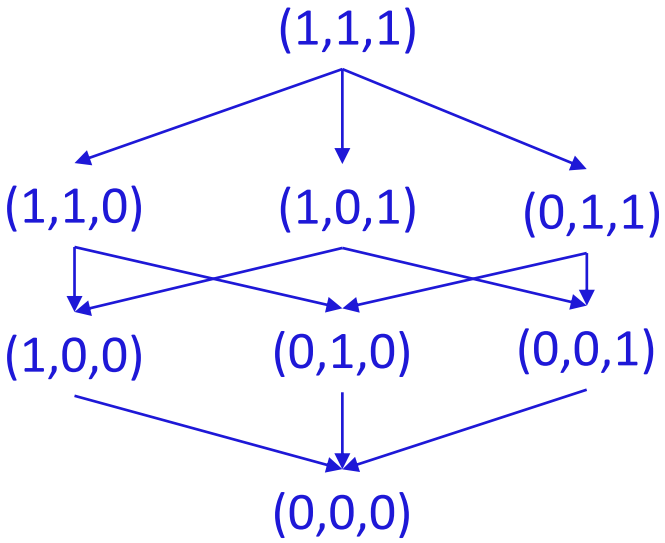
- In many scenarios, many (possibly all) the attributes of interest can assume only one out of a few values (e.g., movies' ratings, presence/absence of a feature, "predicate preferences", domain discretization)

Hotel	Price	Stars	WiFi	Parking	Air Cond.
H1	35 €	*			
H2	30 €	**		✓	
H3	60 €	**			✓
H4	40 €	***	✓	✓	
H5	40 €	**		✓	

- Sky(R) = {H2, H3, H4}, since H2 $\succ$ H1, and both H2 $\succ$ H5 and H4 $\succ$ H5 hold
- The algorithms considered so far are unable to exploit the peculiarities of low-cardinality domains

LS-B: all attributes have low cardinality

- The LS-B algorithm [MPJ07] assumes that all attributes have low cardinality
- Without loss of generality, we consider m Boolean attributes
- The corresponding Boolean lattice consist of 2^m elements, which can be ordered considering that "1 is always better than 0"



- The idea of LS-B is that only tuples in the "best classes" in the lattice are part of the skyline

The LS-B algorithm

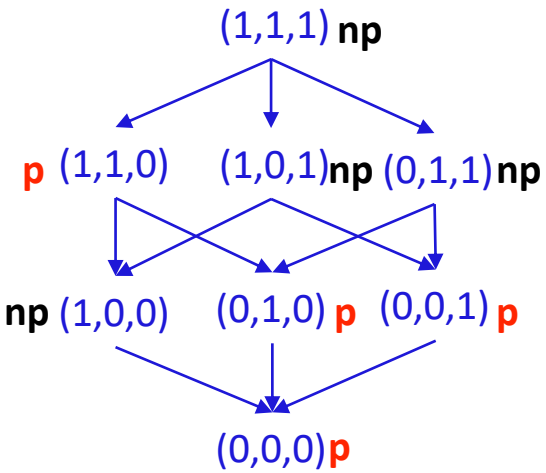
- LS-B operates in two phases:

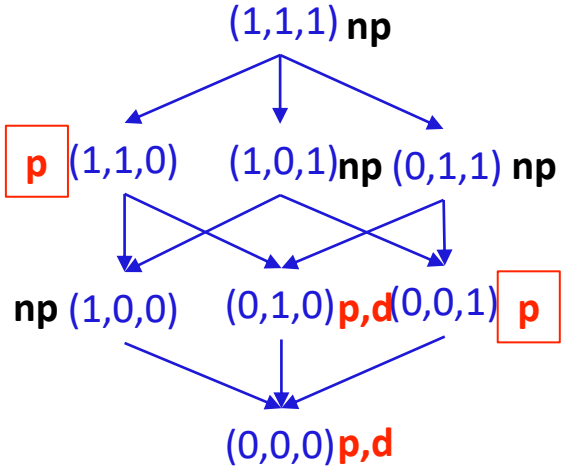
  Phase 1: read all tuples and mark as **present** (**p**) the corresponding elements in the lattice; others remain **not present** (**np**).

  At the end, determine those p elements that are also dominated (**d**)

  Phase 2: read again all tuples and output those whose lattice element is undominated

Hotel	WiFi	Parking	Air Cond.
H1			
H2		✓	
H3			✓
H4	✓	✓	
H5		✓	

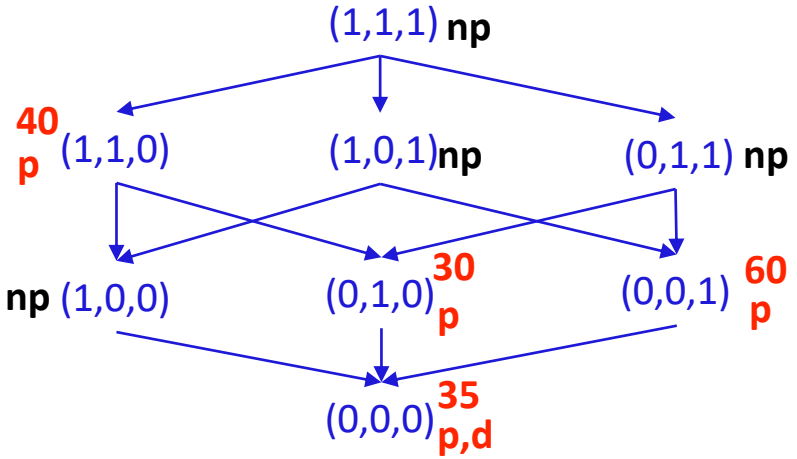




LS: all attributes but one have low cardinality

- The LS algorithm [MPJ07] extends LS-B by allowing the presence of an attribute A0 whose domain can be arbitrarily large (e.g., Price)
- In the 1st phase LS also computes the locally optimal value (**lov**) of A0 for each present element (e.g., the lowest price). An element e is now dominated if there is a better lattice element e' whose lov is no worse than e.lov

Hotel	Price	WiFi	Parking	Air Cond.
H1	35 €			
H2	30 €		✓	
H3	60 €			✓
H4	40 €	✓	✓	
H5	40 €		✓	

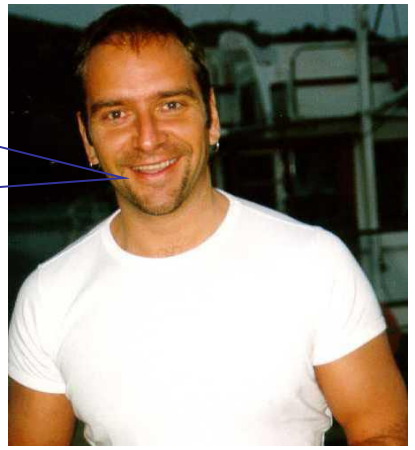


- In the 2nd phase, a tuple t whose element e is undominated can be pruned iff t.A0 is worse that e.lov
- No simple efficient extension is known when more that one attribute has a large domain (for each element we shoud compute a "local" skyline…)

Variants of skyline queries

- [PTF+03] introduces some variants of basic skyline queries:

1. **Ranked skyline queries**
   ranking within the skyline with a scoring function
2. **Constrained skyline queries**
   limiting the search region
3. **K-dominating queries**
   the k tuples that dominate the largest number of other tuples



Dimitris Papadias

- Many other skyline-related problems have been proposed/studied so far, e.g.:
  - Reverse skyline queries: given a query point q, which are the tuples t such that q is in the skyline computed with respect to t (when t is the target)?
  - Representative skyline points: which are the k "most representative" points in the skyline?
- See [CCM13] for a recent survey on the subject

Summary on skyline queries

- Skyline queries represent a valid alternative to top-k queries, since they do not require any choice of scoring functions and weights
- The skyline of a relation R, Sky(R), contains all and only the undominated tuples in R, i.e., those tuples representing “interesting alternatives” to consider
- Computing Sky(R) can rely on both sequential and index-based algorithms
- The BNL algorithm works by allocating a main-memory window, and then comparing incoming tuples with those in the window
- SFS pre-sorts data yielding a topological sort that introduces several benefits compared to BNL
- SaLSa adds a stop condition, that avoids reading all the data
- BBS is a provably I/O-optimal algorithm for computing Sky(R) using an R-tree
- LS-B and LS are designed to work with low-cardinality domains (and at most one large attribute domain)

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